

ALGEBRA of the WEAVING PATTERNS, MUSIC and KINSHIP SYSTEM of the KANKANA-EY

WEAVING TEAM

Teofina A. Rapanut
Ma. Madelyn Ragasa-Cirilo
Reamar Eileen R. Sales

MUSIC TEAM

Henry N. Adorna
Edna M. Nazaire
Reynaldo P. Rimando

KINSHIP TEAM

Wilfredo V. Alangui
Arlano R. Aquino
Avelino P. Bucaoto



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Bureau of Non-Formal Education
Republic of the Philippines
and
Education Research Program
Center for Integrative and Development Studies
University of the Philippines

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**THE ALGEBRA
of the
WEAVING PATTERNS,
GONG MUSIC and
KINSHIP SYSTEM
of the
KANKANA-EY OF MOUNTAIN
PROVINCE**

The Faculty of the Discipline of Mathematics
UNIVERSITY OF THE PHILIPPINES
COLLEGE BAGUIO

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Nanette M. Tolentino, Deputy Program Director

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FOREWORD

U.P. College Baguio is committed to enhancing the UP presence in the Cordillera and Northern Luzon. We practice this commitment in all our teaching, research and extension activities. Much of our research endeavors focus on the Cordillera and the wealth of its indigenous culture, institutions and knowledge.

The Division of Natural Sciences and Mathematics of U.P. College Baguio, particularly the faculty of the Mathematics discipline, deserves commendation for this pioneering study on the indigenous mathematical knowledge of the Kankana-ey, one of seven major ethno-linguistic groups in the Cordillera. Through this research, the faculty researchers of the Discipline of Mathematics have further proven the richness of the indigenous Kankana-ey culture, as they present mathematical abstractions in their customary activities, like weaving and the playing of gongs, as well as in their kinship structure.

Also noteworthy in this research endeavor is the development of formal lessons for use in local schools to demonstrate how practical community activities may be used to enhance the teaching and learning of abstract concepts in mathematics. Thus, this book makes a valuable contribution to the deepening of both our mathematical and pedagogical knowledge.

May this work be only the beginning of the continuing efforts of the U.P. College Baguio faculty to discover how mathematics is embedded in our people's culture.

JESSICA K. CARIÑO
Dean

TABLE OF CONTENTS

ACKNOWLEDGMENT

Chapter 1	INTRODUCTION	1
Chapter 2	THE THEORY OF ALGEBRAIC STRUCTURES	3
	On Weaving Patterns	
	On Music Ensembles	
	On the Kinship System	
Chapter 3	NOTE ON THE RESEARCH DESIGN	11
	The Setting	
	Methodology	
	Limitation of the Study	
Chapter 4	THE ETHNOMATHEMATICS OF THE KANKANA-EY	17
	The Algebra of the Weaving Patterns	
	The Algebra of Gong Music at Weddings	
	The Algebra of the Kinship System	
Chapter 5	TRANSFORMING PRACTICAL KNOWLEDGE TO FORMAL KNOWLEDGE	49
	Lesson 1 : The Set	
	Lesson 2 : Operations on the Elements of Sets	
	Lesson 3 : Properties of an Operation on the Elements of a Set	
	Lesson 4 : Algebraic Structures	
	BIBLIOGRAPHY	93

LIST OF TABLES AND FIGURES

<i>Tables</i>	<i>Page</i>
1 DERIVED PATTERNS FROM THE KANKANA-EY FABRICS	19
2 THE SEVEN FRIEZE GROUPS, THEIR GENERATORS AND THE PATTERNS REPRESENTING THEM	25
3a THE BEAT SCHEME OF <i>TAKIK</i>	32
4a THE BEAT SCHEME OF <i>PINANYOWAN</i>	33
3b THE TRANSFORMED BEAT SCHEME OF <i>TAKIK</i>	34
4b THE TRANSFORMED BEAT SCHEME OF <i>PINANYOWAN</i>	37
 <i>Figures</i>	
1 PHOTOGRAPHED SAMPLE OF EMBROIDERED CLOTHS OF THE PEOPLE OF KUBA, CONGO	4
2 A PATTERN REPRESENTATION OF F_2^2	26
3 A PATTERN REPRESENTATION OF F_2^1	26
4 BASIC DESIGNS OF KANKANA-EY STRIP PATTERNS	27
5 STRIP PATTERNS DERIVED FROM A <i>TIKTIKO</i> DESIGN	29
6 A PATTERN REPRESENTATION OF F_1^1	30

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A Mathematics Discipline of one college, with nine faculty, can do but a little thing. But by uniting our efforts with agencies and institutions, as well as people who realize with us that mathematical knowledge is deeply imbedded in our culture, much can be done for the development of functional literacy programs in our country. We then express our deep gratitude and appreciation to the following:

- The villagers, the municipal and barangay officials, the school teachers and principals in the four Northern Kankana-ey areas who brought us to a deeper understanding and appreciation of their skills in weaving and gong playing, and insights on their kinship system.
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THE RESEARCH TEAM

I

INTRODUCTION

Mathematics is beautiful and essential. It is indispensable to the understanding and improvement of life. However, it is greatly misunderstood and not a few have a natural aversion to it. This aversion stems mainly from its basically abstract nature. It is a basic task therefore of all mathematicians to make their subject matter more accessible to students. This can only be done by relating the abstract mathematical concepts to the real world experiences of the learner. Only then can better understanding and appreciation of mathematics commence.

As part of the natural process of socialization, learners do acquire considerable mathematical knowledge without systematic teaching. This is usually propagated by the existing culture without their realizing that this is mathematics in practice. It is the role of the teacher to be aware of and to highlight these practices and bring them to the level of abstract mathematical learning. This has implications on the traditional teaching-learning perspective of mathematics education, particularly in the Philippines where very few researches have been done in this area of concern. Hence, the basic challenge mathematics education faces is how to teach abstract mathematical concepts in both formal and non-formal instruction within the framework of mathematics in practice.

It is within this framework that the Discipline of Mathematics, U.P. College Baguio undertook this pioneering research to uncover the indigenous mathematical concepts operating in various aspects of

Kankana-ey culture in Mountain Province, specifically weaving patterns, gong music, and kinship system. After several months of continuous study and direct interaction with the Kankana-eyes, the team was able to establish the existence of mathematical structures in these three identified areas of Kankana-ey culture.

2

THE THEORY OF ALGEBRAIC STRUCTURES

One manifestation of the abstract character of mathematics is its being axiomatic. This means that the development of a mathematical concept starts with a set of assumptions on a set of defined objects wherein functional relations are given. This set of assumptions, which are called axioms or postulates, draw the line in making further statements, called theorems, on the set of objects and its relations. Such an axiom set, together with its undefined elements form the basis of an axiomatic system, forming a mathematical structure. In particular, an **algebra** or an **algebraic structure** consists of an arbitrary set, a functional relation on the elements of the set, and a set of rules or axioms on such relations.

For example, the set of all counting numbers, together with the basic operations of addition and multiplication form an algebraic structure. The set of letters in the English alphabet, however, has only a faint semblance of mathematical structure because the letters are 'ordered', that is, A is followed by B, which in turn is followed by C, which is followed by D, and so on. The order is arbitrary, and is so since the users of the language have accepted it by agreement and custom.

The principles of symmetry, regularity and order seen in the weaving patterns, gong music and kinship system suggest the existence of an underlying mathematical structure about which even the weaver,

gong player and an “elder” in the community may only be partly conscious or unaware.

On Weaving Patterns

The Kankana-ey fabrics have been found to be predominantly longitudinal striped materials, with iconographic motifs so organized as to form a system of well-defined formal relationships. Such an organization is essentially a repetition of a motif in one direction by the application of certain geometric plane motions which preserve distance. The aggregate of figures so generated possesses form and symmetry, and is referred to as a **frieze pattern**.

The motions which generate such a pattern are referred to as **geometric transformations**. The structure which these transformations form is called a **frieze group**.

The repetitive patterns in decorative arts have been found to belong to one of the 7 types of frieze patterns, or to the 17 types of those which repeat in two directions, called the **wallpaper patterns**. These classifications were done notably by Polya and Niggli in 1924 and by Speiser in 1927. Figure 1 shows photographed samples of embroidered cloths of the people of Kuba, Congo in which several types of frieze patterns can be seen.

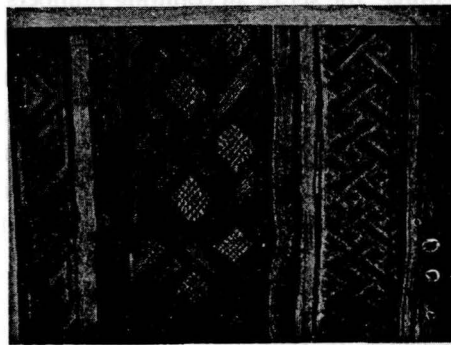


Figure 1. Photographed Samples of Embroidered Cloths of the People of Kuba, Congo

It is in Arabic and Moorish designs that the different kinds of symmetry have been most fully explored, perhaps because of its non-representational style. Grunbaum, Grunbaum and Shepard (1986) investigated the Moorish ornaments in Spain and found patterns that belong to 13 wallpaper classes. Moreover, they are found to be symmetric in several different ways.

Zaslavsky (1973), on the other hand, observed that there are many examples in African art of patterned strips, as in a carved wood door panel in eastern Yoruba, Nigeria, or as borders on cloth or carved objects. More often than not, the particular pattern has a name and some significance in the lives of the people. The repetitive patterns of African art have also been analyzed mathematically and have been classified according to the seven types of frieze patterns.

Milgram (1991) discussed that to “understand more fully the significance of artifacts within their cultural context, one must examine both the physical qualities of the object, its materials and design, and its patterns of movement, its circulation and distribution.” Milgram analyzed the cloth technology and design structure of the Highland Luzon textiles covering the cultural groups of Ifugao, Kankana-ey, Bontoc, Kalinga and Gaddang. Formally trained in weaving, she gave a detailed description of the textile production and trade both from the historical and ethnographic contexts. She made an “analysis of the design structure, concentrating on the organization of the formal elements by symmetry motions, which are the objective and universal criteria to determine the rules that weavers follow in organizing their woven compositions.”

Milgram’s analysis of the Cordillera textiles indicates that “tripartition is a major principle in the following categories: i) the physical construction of the cloth, ii) the placement and structure of the weaving techniques, and iii) the colour scheme”. Another principle used is the existence of biaxial mirror reflection, using two central axes, vertical and horizontal, which yield two symmetrically opposed spaces, top/bottom, right/left of the fabric.

Milgram analyzed the Cordillera woven fabrics “according to the spatial organization of its design elements manifesting relationships between its parts: right/left, center/sides, top/bottom, and those between the parts in relation to the whole”.

This study, on the other hand, analyzes the Kankana-ey fabrics by classifying the woven patterns according to their symmetry groups. Because art styles are visual, and are readily grasped, the study of such symmetry groups can establish the presence of intuitive concepts of symmetry based on mathematical principles among the Kankana-ey weavers.

On Musical Ensembles

Kankana-ey music is usually produced by gongs and other instruments which produce sound by beating them. The regularity of the beating of each instrument results in a distinct **beat pattern**. The totality of these beat patterns brings out the **beat scheme** of the ensemble played by the instruments. Some studies have shown that the regularity present in musical arrangements exhibits an underlying mathematical structure.

Henri Martin (1984) gave a brief review of the numeric character of the most popular musical scales and suggested that there is a connection between successful musical composition and the mathematics of symmetry and proportion. Martin discussed three reasons why the connection is very close: first, the effect of a musical sound upon our ears depends first and foremost on its pitch, or frequency in physical terms. To say that we hear middle c is to say that our ear registers 256 vibrations per second. Thus, a number, integer or real, is associated with sound. Second, hearing two sounds simultaneously is equivalent to perceiving two numbers and their relationship, i.e. ratio. Experience with music shows that the aesthetic effect of a chord depends almost exclusively on the ratio of frequencies; the whole question of harmony is therefore a question of the choice of ratios. Third, rhythm by nature is essentially arithmetical.

Most of the studies which relate mathematics with music were done by studying the physical characteristics and qualities of the sound phenomenon. Each sound has several dimensions, the more fundamental of which are pitch and duration (Mirano, 1988). Any melody that we encounter is a fusion of pitch and duration (1971). By eliminating the element of pitch from the melody, a durational pattern is produced. The pattern produced by any piece of music is what inspired people to look for ways of interpreting them mathematically. But since in the creation of sound the basic requirement is the production of a definite pitch or tone, pitch therefore has become very useful in the course of analysis.

Robert Donington, in *The Instruments of Music*, also discussed how to interpret sounds mathematically. Donington, for example, dealt with the sound produced by a tuning fork. When this apparatus is struck by something hard, sounds are produced and curves can be recorded which plot the behavior of the sound vibrations. These wave-like outlines of the curves depict the extent and relationship of the displacements occurring in the sound-vibrations. Once this is understood, we say we have a graphic representation of what is really happening.

Budden (1972) discussed the existence of group structure elements in the musical pieces of Bach and other classical music. The regularity in these pieces is apparent on the kind of musical arrangement.

Our survey of published materials available to us showed that although there are several methods of analyzing sounds mathematically, none of these methods was used to describe the mathematics of gong music. In fact, to our knowledge, there has been no work yet that analyzed the beat pattern of gong music. This may be due to the fact that, unlike other instruments which produce a pitch of different frequencies, a gong produces only one distinct pitch, the frequency of which depends on the size of the gong. This study considers the manner by which each instrument is struck to come up with its beat pattern.

On the Kinship System

Kinship is a study of the relationships of people, especially those relationships which are products of marriage and descent. Social groupings exist within any society, where the members follow certain norms and rules that dictate the manner by which they relate with each other. We can therefore think of a society as a formal organization where each individual has a defined role in relation to other members of that society. For instance, a person relates differently towards a relative (either by blood or marriage), towards a “friend”, and towards an ordinary person, or a stranger. Hence the norms or rules are there to maintain order within the social organization. It has long been accepted by anthropologists that kinship systems may exhibit a **group structure** that may help explain the principles underlying order within the society.

The following definitions are due to Keesing(1975). **Kinship system** is the “whole conceptual and social field relating to kinship, marriage, and descent.” **Kinship** refers to “connections between parents and children (i.e. connections of filiation) and to the networks of relationship built out of these parent-child links.” **Descent** is a relationship defined by “connection to an ancestor (or ancestries) through a culturally patterned sequence of parent-child links.”

Andre Weil unlocked the door to kinship mathematics. In 1949, Weil showed a way to analyze certain kinship systems using algebra. In particular, he applied the theory of groups of permutations to certain types of marriages.

Later, in an undated paper, R.R. Bush proposed making use of permutation matrices. Permutation matrices proved to be a more convenient tool in the analysis, subsequently adopted by Kemeny, Snell and Thompson when they developed Weil’s ideas. They formulated eight important axioms that focused on the types of marriages found in certain societies.

Harrison White (1963) expanded the Kemeny-Snell-Thompson work in *An Anatomy of Kinship*. White carried the analysis in terms of clans and not in terms of marriage types as done by Weil, Bush, and

Kemeny, Snell and Thompson. Anthropologists and people, White noted, do not naturally consider marriage types; rather, anthropologists and people think more naturally in terms of social groupings, e.g. clans, kindreds and relatives.

White showed that when kinships are investigated in terms of clans, the mathematical formulations and derivations are simpler and results are easier to interpret. White started by reformulating the eight axioms of Kemeny, Snell and Thompson.

From the eight axioms (see Appendix G), White identified two permutation matrices. One matrix represents the transformation of the husband's clan into the wife's clan. The other matrix represents the transformation of the father's clan into the children's clan. These two permutations generate a group structure. This abstract group corresponds to at least one possible society. The same abstract group, White proved, can represent not only distinct societies but different types of societies as well.

White's work was a breakthrough in kinship mathematics. Nevertheless, his model fits and describes only those societies with unilineal descent systems. It fails to characterize societies with bilateral systems. In fact, it cannot characterize any society that fails to meet the qualifications embodied in the eight axioms mentioned above.

A review of the literature available to us revealed the absence, or lack of, mathematical studies on bilateral kinship systems such as in the Philippines where kinship is reckoned from both the maternal and paternal sides (therefore bilateral as opposed to unilateral). This could be because of the generally lukewarm treatment that bilateral kinship systems received from scholars. As Keesing (1975) put it, "societies that are not unilineal were long relegated by anthropologists to a negative leftovers bag of bilateral or cognatic systems." In Jocano's *Sulod Society* (1968), he stated with an air of frustration, that "Anthropologists ... have largely ignored societies which have bilaterally structured kinship systems despite the fact that the reckoning of relationship with both the mother's and the father's kin group (i.e. bilateralism) is found throughout the world."

Our inability to use White's model in the analysis of the Kankana-ey kinship system is based on the contention that there are no clans in the Philippines. In arguing this, Panopio and Rolda (1988), explained that the characteristics of a clan are not found in any of the kinship groupings in the country. According to Panopio and Rolda, "a clan exists in a group whose line of descent is unilineal, whereas all Philippine groups are bilateral." How then do we explain the Filipinos' tendency to refer to their grouping as a clan? Panopio and Rolda explain thus:

Basically, the reason why Filipinos use the term *clan* to refer to their kind of group is that a large number of relatives surround the individual. Instead of a clan, a more appropriate term is *kindred*.

3

A NOTE ON THE RESEARCH DESIGN

This study dealt with the Kankana-ey, a cultural group in the Cordillera. Various documents use "Kankanay", "Kankanai" or "Kankana-ey" to refer to this people. We opted to use "Kankana-ey" because it is closest to the way the informants pronounce their tribal name. We also use Kankana-ey not just to refer to the people, but to their language and culture as well.

The Kankana-ey comprise the third largest cultural group in the Cordillera, after the Ifugao and Bontoc cultural groups. There are two Kankana-ey subgroups: the Northern Kankana-ey of Mountain Province and the Southern Kankana-ey of Benguet. The northern Kankana-ey, known as Lepanto *Igorot*, *Aplai* or *Iyaplay* inhabit the municipalities of Besao, Sagada, Tadian, Bauko and Sabangan. The southern Kankana-ey, meanwhile, inhabit the municipalities of Mankayan, Bakun, Kibungan, Buguias and the upper half of Kapangan.

We chose to focus on the Northern Kankana-ey of Mountain Province because the communities of this subgroup have persistently preserved their indigenous culture. To this date, they are essentially governed by centuries-old traditions and practices. The elders or *lallakay* still play definitive roles in the conduct of various rituals, such as in weddings, wakes and burials, and pre- and post-harvest. The old folks, however, comprise less than 10 percent of the population and their number is dwindling. The young comprise more than 90 percent of the

population and they have been exposed to modern influence by their education in urban areas. This situation explains why the community rituals are slowly being modified.

We then concentrated on the weaving patterns, music and kinship system of the Kankana-ey. (From here onwards, we use Kankana-ey only to designate the Northern Kankana-ey.) These three features of the Kankana-ey culture are present in many important community activities. In the wedding, for instance, women wear the traditional skirt called *tapis*. Men play the brass gongs and the *solibao* and this dominates the musical atmosphere. The couple's kin and kindreds grace the celebration and they listen to the *lallakay* enumerate completely the genealogy of the new couple.

This study is not a comprehensive anthropological documentation on these three aspects of Kankana-ey culture. Instead, this is a mathematical analysis of the weaving patterns, gong music and kinship system with an objective of finding algebraic structures in them.

The Setting

There are four Kankana-ey areas covered by this research, namely; barangay Guinzadan, Bauko; barangay Data, Sabangan; barangays Kin-iway and Banguitan, Besao; and barangay Demang, Sagada.

All four areas are basically rice-producing communities, and the rice they produce is mainly for personal consumption. The people of Guinzadan, however, are slowly turning to vegetable farming as a source of livelihood, while a significant number of people in Demang are engaged in small-scale businesses, it being a tourist spot. Weaving of fabrics is an economic activity of women during off seasons of planting.

These areas can be reached through rugged and dusty roads which are vulnerable to landslides and soil erosions during the rainy season. This is especially true in Guinzadan and Data.

There is one important point that has to be made regarding the four sites: they are all being subjected to, and are undergoing culture changes. The influx of foreign and local tourists, as well as the interaction of Kankana-ey students in urban centers of learning, have slowly manifested their influence on their indigenous culture. However, serious efforts are being made to retain certain important aspects of the traditional culture.

Methodology

Since the traditional features of weaving patterns, music and kinship system are the focus of this study, the key informant method in gathering the data was used. The informants were the *lallakay* (old men) and the *babbaket* (old women). These are the elders of the community. These elders are regarded by the community as the source of reliable information and respected authority, especially on kinship matters. They are regarded as the keepers of the lore and traditions of the community. They are expected to define correct customs, to interpret procedure in *canao*, and remember significant matters from the past. To identify these elders, the help of the barangay captains and some residents was solicited.

In the case of the weaving component, the team interviewed those who still use the traditional *pinnagud* or backstrap method and own samples of woven fabrics as part of the family heirloom. The respondents were initially asked to identify the traditional fabrics woven by the Kankana-ey women and to state the local names of the different types of these fabrics. Photographs of the named sample fabrics were then taken. Strip patterns, that is, patterns which extend in one direction were then derived from these photographs, and were further drawn using computer graphics. The weaving team had to cross check the computerized strip patterns with the actual weaving patterns on the sample fabrics. The research assistant had to stay in one community for a month not only to do this, but also to understand and learn how to weave. The computerized patterns were then analyzed in order to classify them according to the seven established types of strip or frieze patterns.

The music team, on the other hand, attended three *babayas* or wedding feasts during the data gathering stage so that the gong music as played during the wedding ritual, can be recorded. Moreover, the team looked for trained gong players in the communities to serve as the control group. The group played purposely for the music team. Thus, the recording of the gong music was made clearer, since some sounds which come from the crowd's greetings and conversations during the actual wedding ritual, were eliminated. These gong players who are pupils of the Banguitan Elementary School, were also considered as key informants, and the participatory interview method was utilized. The playing of each instrument was recorded separately, and the music team members tried to play the gongs after listening to the recorded music. The teacher-trainer of this control group was asked to check on the team members' playing of the gongs. An ethnomusicologist was also asked to transcribe the recorded music. The analysis of the recorded and transcribed music was done to identify the beat pattern (the regularity of the beating) of each instrument, as well as to determine the beat pattern produced as two or more instruments are played together.

For the kinship team, guide questions were formulated to determine the key informants' concept of kinship and descent. Discussions and interviews were non-structured and free flowing. The team also gathered the local terminologies used for kinship and descent during these conversations and interview.

Limitations of the Study

This preliminary investigation considered only patterns obtained by repeating a motif in one direction from fabrics which have the traditional Kankana-ey patterns. Other types of patterns like those which repeat in two directions were not included.

This study focused only on the gong music played during the wedding ritual. Initially, it considered three kinds of music produced by gongs, namely, **Takik**, **Pinanyowan** and **Balangbang** but the recorded music of the Balangbang did not include sounds produced by a pair of gongs called *elwas*. Thus, only the **Takik** and **Pinanyowan** Ensemble were analyzed.

White's model of analysis of a kinship system which uses **clans** as basis cannot be used with the Kankana-ey society. A clan uses a unilineal line of descent but the Kankana-ey kinship system is bilateral. Because of this study's preliminary nature and the absence of previous studies in the literature, it was decided to limit the study to algebraic structure in two features of the Kankana-ey kinship system, namely, their descent grouping and kindred grouping.

4

THE ETHNOMATHEMATICS OF THE KANKANA-EY

D'Ambrosio (1984) has used the expression “ethnomathematics” to refer to forms of mathematics that vary as a consequence of being embedded in cultural activities whose purpose is other than “doing mathematics.” The current interest in ethnomathematics around the world made us look for indigenous activities in the Cordillera Region which may manifest some mathematical concepts. Weaving, playing of gongs and the relationship of peoples are activities which are embedded in the Cordillera culture yet mathematical abstractions, with their intricacy and sophistication, are noteworthy in them.

In this chapter we examine the data gathered in the Kankana-ey weaving patterns, gong music and the kinship system in order to establish the existence of algebraic structures.

The Algebra of the Weaving Patterns

The weaving team gathered two sets of data: 1) the responses to the guide questions on weaving as one of the Kankana-ey women's preoccupation and as one of the features of their culture, and 2) the photographs of the Kankana-ey fabrics from which patterns were derived for classification according to their symmetry groups.

According to function, there are six kinds of fabrics which the Kankana-ey women weave, namely,

- a blanket or *ules*;
- woman's skirt or *tapis*;
- woman's waistband or *bakget*;
- man's loincloth or g-string or *wanes*;
- man's headband or *bedbed*; and
- a decoration.

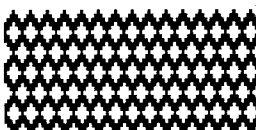
Only the first four of these fabrics are considered in this study because these are what majority of the traditional weavers produce and those of which the informant weavers have shown samples. Appendix C shows the photographs of the sample fabrics labeled according to their function (whether *ules*, *tapis*, *bakget* or *wanes*). Moreover, the local names of each sample fabric in each research site is indicated.

Note, however, that a picture of a type of *ules* is indicated to represent also a type of *tapis*. This was confirmed by the responses of the informants that the *ules* and the *tapis* have the same design elements which determine the local name of the blanket. This study considered only four designs for *ules* or *tapis*, namely *pinaggpagan*, *kinayan* (*lineyeg* or *lanlan*), *kinain* or *kinaut*, and *innangin*. One type of *ules* or *tapis* which the informants claim bears a recent design is called *kinulibangbang*. This type, which is represented by Picture No. 3 in Appendix C has the *pinaggpagan* designs in the middle panel of the fabric and the *kulibangbang* designs in the side panels.

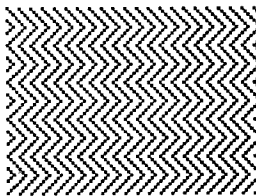
On the other hand, the variations of design elements for a *bakget* are *golingán* (*sinapuan* or *binontoc*) and *ginaspala* (*inawing*), while for that of a *wanes* it is *binuldaán* (*sinulaman* or *binnalikes*).

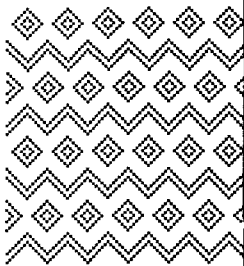
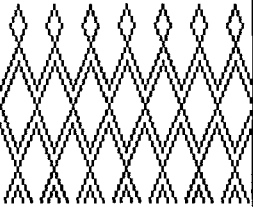
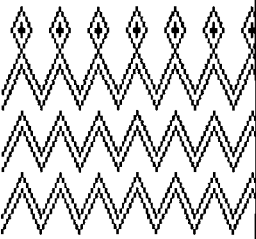
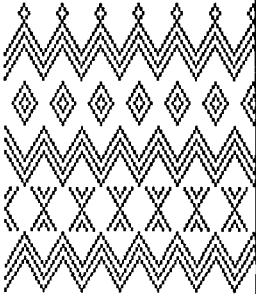
The weaving team derived twenty-eight (28) distinct patterns from the photographed fabrics and these are summarized in Table 1. Such patterns are those which are formed by the repetition of some design or motif either horizontally or vertically on the fabric. The table also includes the kind and type of fabric where the pattern is derived.

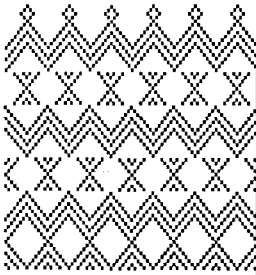
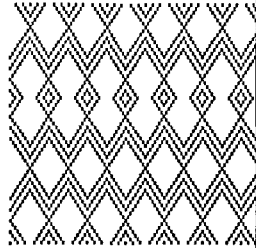
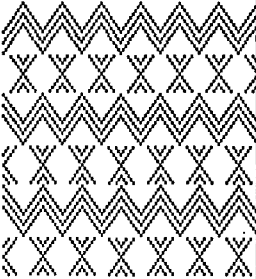
Table 1
DERIVED PATTERNS FROM THE KANKANA-EY
FABRICS

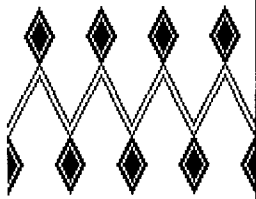
PATTERNS	<i>ULES</i>	<i>TAPIS</i>	<i>BAKGET</i>	<i>WANES</i>
#1 	Ginawa/ Innangin/ Timken	Ginawa/ Innangin/ Timken		
#2 	Lineyeg/ Lanlan/ Kinayan	Lineyeg/ Lanlan/ Kinayan		
#3 	Pinag- pagan	Pinag- pagan	Sinapuan/ Golingán/ Binontoc	
#4 	Pinag- pagan	Pinag- pagan		
#5 				Binul- daan <i>or</i> Binna- likes <i>or</i> Sinula- man

#6		Pinag-pagan	Pinag-pagan		
#7		Pinag-pagan	Pinag-pagan		
#8		Kinuli-bangbang	Kinuli-bangbang		
#9		Kinuli-bangbang	Kinuli-bangbang		
#10		Kinain	Kinain		
#11		Kinuli-bangbang	Kinuli-bangbang	Golingan/ Sinapuan	

#12		Kinuli-bangbang	Kinuli-bangbang	Golingan/ Sinapuan/ Binontoc	
#13		Kinain/ Kinaut	Kinain/ Kinaut		
#14			Kinain		
#15				Golingan or Sinapuan or Binontoc	
#16				Golingan or Sinapuan or Binontoc	
#17				Ginaspala or Inawing	

#18 	Kinain or Kinaut	Kinain or Kinaut		
#19 				Binna- likes <i>or</i> Binul- daan <i>or</i> Sinu- laman
#20 				Binul- daan or Sinu- laman
#21 				Binul- daan <i>or</i> Sinu- laman

#22 				Binna- likes or Binul- daan or Sinu- laman
#23 				Binul- daan or Sinu- laman
#24 				Binul- daan or Sinu- laman
#25 	Pinag- pagan	Pinag- pagan		

#26		Pinag- pagan	Pinag- pagan		
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#27			Pinagpagan	Pinagpagan	

					
#28			Pinagpagan	Pinagpagan	

The derived patterns were classified according to the seven frieze group structures. Our mathematical discussion of the frieze groups is found in Appendix B. For the analysis of such data, however, such groups are described through the patterns that they generate. Table 2 shows the seven frieze patterns using the motif R, including the generators of that group which each pattern represents.

Pattern No. 1 is a part of the white strips woven over the black vertical stripes in the white blanket exhibited in Picture No. 2 (Appendix C). This pattern is classified as F_2^2 . Figure 2 shows the motif and the generators of the group, which are a reflection about the line l and a half-turn about the point A .

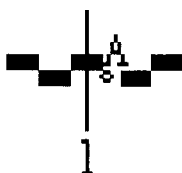


Figure 2. A Pattern Representation of F_2^2

On the other hand, Pattern No. 2 is a yellow and green strip woven over the white vertical stripe of an *ules* or *tapis* called *lineyeg*, exhibited in Picture No. 1. The pattern is classified as F_2^1 , verified in Figure 3 which shows the generating motif and the generators, which are reflections on the lines l_1, l_2, l_3 .

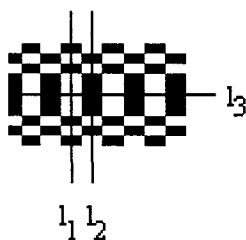


Figure 3. A Pattern Representation of F_2^1

The Kankana-ey strip patterns use four different designs, namely *matmata*, *tiktiko*, *sopo* and *kulibangbang*. Figure 4 shows the basic motif for each of these designs. Variations of each design are also found in the figure. These designs and their variations have been combined to produce the beautiful Kankana-ey strip patterns.

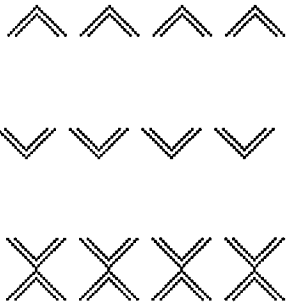
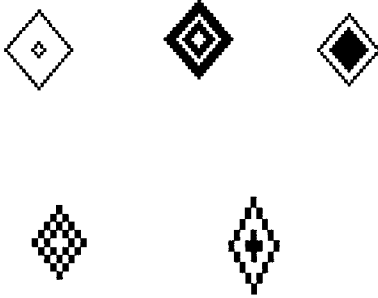
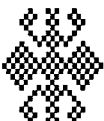
Basic Design		Variations
Tiktiko		
Matmata		
Sopo		
Kulibangbang		

Figure 4. Basic Designs of Kankana-ey Strip Patterns

Most of the informant weavers claim that such designs are representations of objects the people see in their environment. They claim that the tiktiko represents the mountain foot trails as well as the zig-zag mountain roads, while the mat-mata represents a bird's eye. The sopo represents a flower while the kulibangbang represents a butterfly. Some weavers claim that the V-shaped variation of the tiktiko represents a fish trap in the river while the X-shaped variation of the tiktiko represents a mortar. The weaving techniques used to produce these designs are discussed in detail by Milgram.

The tiktiko is woven either in a single row in various colors - green, yellow, red, blue or pink - as in the *bakget* in Picture No. 20 in Appendix C. Such a strip pattern is classified also as F_2^2 . Figure 5-a shows why this is so. However, the tiktiko may occur in 4 to 8 horizontal rows (see Figure 5-b) as woven in the *pinaggagan* blankets in Picture Nos. 4, 5 and 8 in Appendix C. Pattern No. 3 in Table 1 illustrates this. As such, the strip pattern is classified as F_1^2 . Another arrangement made is to have several rows of tiktiko with their tips of each row coinciding; thus forming small diamonds which also come in rows, as seen in Pattern No. 4 in Table 1. This pattern is classified as F_1^2 if there is an even number of rows of diamond (see Figure 5-d) and it is F_2^1 if it is odd (see Figure 5-c).

The tiktiko may be used as borders of patterns using other designs as shown in Pattern Nos. 6, 7, 15 and 16 of Table 1. This combination represents a frieze group which is based on the bordered design. Let us recall that the single row of tiktiko represents an F_2^2 . Pattern No. 5 of Table 1, which is a design alternating a double matmata with the mortar-like design, is classified as F_2^1 , while Pattern No. 6, which is essentially Pattern No. 5 bordered on both sides by a single row of tiktiko, is classified as F_2^1 . Similarly, Pattern Nos. 7, 15 and 16 of the same table are classified as F_2^1 .

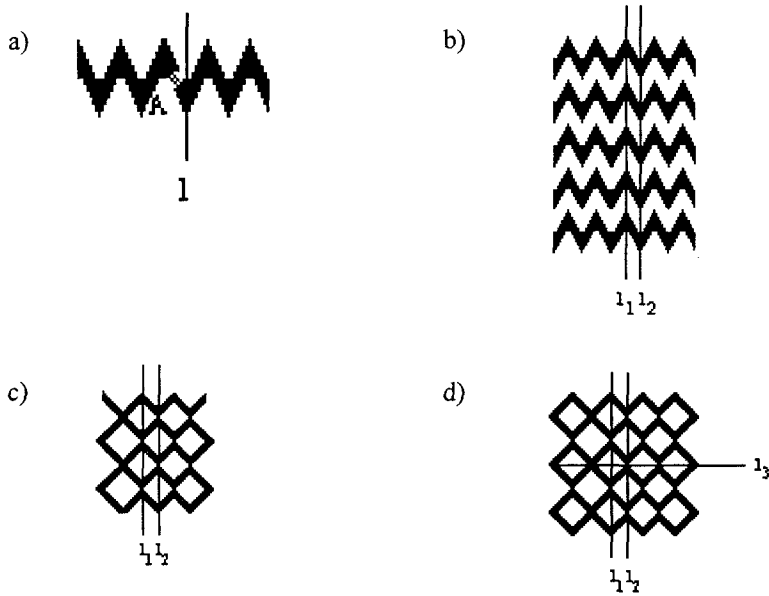


Figure 5. Strip Patterns Derived from a Tiktiko Design

Another variation of *tiktiko* is its having been woven with doubled rows as in Pattern Nos. 19, 20, 21 and 27, or trebled rows as in Pattern Nos. 22, 23, 24 and 25. It is interesting to note that such variations combined with any of the variations of *matmata* and/or with the mortar-like design produce a strip pattern which represents the frieze group F_1^2 . Such patterns are generated by two vertical reflections, similar to those indicated in Figure 5-b.

The *tiktiko* is also oriented vertically in a *bakget* called *ginaspala*, as shown in Pattern No. 17 of Table 1. This is the only Kankana-ey pattern classified as F_1^1 . Figure 6 shows how a motif is generated by a reflection on a line l and a translation t to form a part as well as the whole pattern.

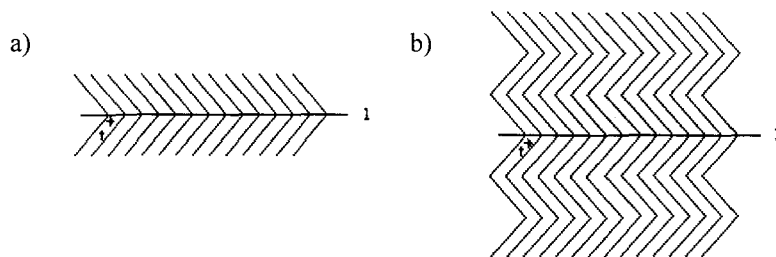


Figure 6. A Pattern Representation of F_1^1

It is worthwhile noting that the matmata, the sopo and the kulibangbang have biaxial symmetry. This means that such motifs are formed by reflections on a vertical and horizontal axes of a part of the motif. Thus, when these motifs are repeated in one direction as in Pattern Nos. 8 to 13, they form a strip pattern which represents the frieze group F_2^1 . Let us recall that this group is generated by two vertical and one horizontal reflections. Pattern Nos. 14 to 16, which alternate a variation of sopo and a matmata separated by one or two vertical bars, are also classified as F_2^1 .

Pattern No. 27 is found in the two side panels of a pinagpagan, a kinain or kinaut. This is oriented vertically along the length of the fabric using black and white threads as seen in Picture Nos. 5,6,7,10,11 and 12, having Pattern No. 28 as its variation. Note, however, that the strip has a snake and man figures symmetrically oriented at the middle. That is the reason that the whole strip does not represent a frieze group. However, if the snake and man figures are removed, the remaining strip is a pattern representing the frieze group F_2^1 .

A complete listing of the frieze group classification of derived Kankana-ey patterns is done in Appendix D.

The Algebra of Gong Music

In this section we describe two types of gong music, namely the *Takik* and the *Pinanyowan*.

The **Takik**, as a musical ensemble, is played with five (5) instruments namely, *solibao*, *takik*, *atomba*, *pingsan* and the *sonob*. The *solibao* is a drum made of a hollow wood or small trunk of a tree, covered on both ends with the skin of a deer. This cover is tapped by hand to produce the required sound. The *takik*, as an instrument, is an iron clapper played with a stone. These two instruments are most essential in the **Takik** ensemble. There can be no ensemble in the absence of any of the two. The *atomba*, *pingsan* and the *sonob* are all gongs.

In playing **Takik** (the musical ensemble), the *solibao* is played first to set the rhythmic mode of the ensemble. After the mode has been established, the other instruments follow: one, two or all at the same time. Usually, it is the *takik* (the instrument) which joins the *solibao* first.

Takik Ensemble is played following a **beat scheme** defined here as the totality of all the beat patterns each instrument is creating. A **beat pattern** consists of beats repeated at regular intervals. Its basic unit is a cluster consisting of four beats.

Table 3a describes how the **Takik** music is produced where the first column lists the instruments involved and the sequence of numbers opposite each instrument indicates the basic unit of its beat pattern.

Table 3a
The Beat Scheme of *Takik*

INSTRUMENT	BEAT PATTERN
Solibao	1 2 3 4
Takik	5 6 7 8
Atomba	5 6 7 [8]
Pingsan	[5][6][7]8
Sonob	improvisation

Solibao is assigned the sequence 1 2 3 4 because it is the lead instrument. This sequence means that it is played first to set the basic rhythmic mode of the ensemble. Only after the basic rhythmic mode has been established can any other instrument be played, at the start of any other cluster. This explains the use of the sequence 5 6 7 8 for the other three. *Solibao* and *takik* are struck at every beat of the cluster, indicated by 1 2 3 4 and 5 6 7 8, respectively. The *atomba* is struck at every first three beats indicated by 5 6 7 [8], while *pingsan* is at every fourth beat indicated by [5] [6] [7] 8. This means that each instrument is not struck at a beat enclosed by []. Once each instrument is played, it continues to be played until the music is finished.

We shall call these four instruments, the regular instruments of the *Takik* ensemble.

The other instrument in the ensemble, the *sonob* does not have a beat pattern. It creates an improvisation and acts as the soloist in the ensemble, yet it still follows the overriding rhythmic mode established by the *solibao*.

On the other hand, *Pinanyowan* or *Palakis* is played with four gongs. These gongs are called *tambol*, *toktok* or *togtog*, *piyak* and *sonob*. The *tambol* is called such because it is played like a drum.

The beat scheme of the *Pinanyowan* is similar to that of the *Takik*.

The *tambol* has a beat pattern similar to that of the *solibao*, but the first three beats are dampened while the last is left to vibrate. The *piyak* has the same beat pattern as *tambol* and it creates the rhythmic mode of the **Pinanyowan**. The *sonob* and the *tok-tok* skip the first beat and continues this pattern throughout the ensemble.

Table 4a
The Beat Scheme of **Pinanyowan**

INSTRUMENT	BEAT PATTERN
Piyak	1 2 3 4
Toktok	[5] 6 7 8
Sonob	[5] 6 7 8
Tambol	5 6 7 8

Again the sequences 1 2 3 4 and 5 6 7 8 opposite *piyak* and *tambol* indicate that both instruments are struck at each beat of every cluster, with the *piyak* played first and the *tambol* played at the start of any other cluster. The sequence [5] 6 7 8 indicates that *sonob* and *tok-tok* are struck at the last three beats of each cluster.

Note that for **Pinanyowan**, *piyak*, *tok-tok*, *sonob* and *tambol* are the regular instruments.

To understand the mathematical structure found in gong music, we shall transform the two tables of beat patterns above. As mentioned earlier, the basic unit of a beat pattern consists of four beats. For purposes of representation, we shall assign either a value 0 or 1 to each of these four beats, hence transforming a beat pattern into a sequence of 0s and 1s. We shall call this sequence of 0s and 1s a **beat code**. Each instrument therefore has a distinct beat code.

A. The *Takik Ensemble*

For the *Takik ensemble*, the beat produced whenever *only* the *solibao* instrument is struck, is assigned the value 0. Hence, the beat code of *solibao* is 0000. The beat code of the other instruments will then

be obtained as follows : at the beat where an instrument is struck simultaneously with the *solibao*, we assign 1; otherwise, we assign 0 (which means that at that beat, only the *solibao* is struck). Here we see that the assignment of 0 or 1 is based on whether an instrument is struck together with the *solibao* or not. We then transform Table 3a as follows:

Table 3b
The Transformed Beat Scheme of *Takik*

INSTRUMENT	BEAT CODE
Solibao	0 0 0 0
Takik	1 1 1 1
Atomba	1 1 1 0
Pingsan	0 0 0 1

For example, what is meant by *atomba* having a beat code 1110? This means that the *atomba* is struck simultaneously with the *solibao* in the first three beats, hence the three consecutive 1s in the beat code. At the fourth beat, the person playing the *atomba* does not strike the instrument so that only the *solibao* is struck and heard at that precise moment; hence the entry 0 in the beat code of the *atomba*.

The beat pattern which reflects the regularity of striking each instrument, creates a rhythm that is unique to that instrument. For this reason, we now associate to each beat code the distinct rhythm that is produced by each instrument.

Let C_T be the set consisting of the beat codes in the ensemble, that is, $C_T = \{ 0000, 1111, 0001, 1110 \}$. It is important to note that each beat code represents the distinct rhythm produced by every instrument. Thus, the beat codes of two instruments are equal if the rhythms of these instruments are basically the same. Also, equal beat codes (i.e. same rhythm) do not necessarily come from the same instrument. For example, the simultaneous playing of the *atomba* and *pingsan* produces a rhythm similar to that of *takik*, that is, the rhythm reflected by a beat code where an instrument is struck at every beat (relative to the *solibao*).

We define an operation v_T on our set C_T . Assume that one instrument is being played and then joined by another instrument. In a sense, the rhythm of the first instrument is now combined with the rhythm of the second, creating some rhythm. The operation v_T is therefore just the combination of two rhythms, joined two at a time. In terms of beat codes, we have $a_1a_2a_3a_4 \ v_T \ b_1b_2b_3b_4 = c_1c_2c_3c_4$, where $a_1a_2a_3a_4$ represents the rhythm of the first instrument, $b_1b_2b_3b_4$ that of the second, and $c_1c_2c_3c_4$ the beat code representing the rhythm produced. We shall call v_T the **join operation**.

The ensemble starts with the playing of the *solibao*, to set the basic rhythmic mode. Let us look at what will happen if only one instrument is played together with the *solibao*. For instance, if *takik* alone follows *solibao*, the distinct rhythm of the *takik*, which is reflected by a beat code where the instrument is struck at every beat (relative to *solibao*) stands out. In this case, the *solibao* serves as an accompaniment. In terms of beat codes, the join of 0000 and 1111 will produce 1111, i.e. $0000 \ v_T \ 1111 = 1111$. The same thing can be observed if either the *atomba* or *pingsan* is played with the *solibao*. In terms of beat codes, we then have:

$$0000 \ v_T \ 1110 = 1110$$

$$0000 \ v_T \ 0001 = 0001$$

Next, let us assume that two instruments at a time joined *solibao*, say *takik* and *atomba*. Notice that both instruments are struck at their first three beats, thus we expect to hear the sound at the first three beats louder than the sound at the fourth beat when only the *takik* is struck together with the *solibao*. Listening to a series of clusters of this sound is like listening to the rhythm the *pingsan* creates, and this is because only at the last beat do we hear a distinct sound. If we try the other combinations: *takik* and *pingsan* or *atomba* and *pingsan*, we obtain a rhythm similar to *atomba* or *takik* respectively. Again, using beat codes, we express this as follows:

$$0000 \ v_T \ (1111 \ v_T \ 1110) = 1111 \ v_T \ 1110 = 0001$$

$$0000 \ v_T \ (1111 \ v_T \ 0001) = 1111 \ v_T \ 0001 = 1110$$

$$0000 \ v_T \ (1110 \ v_T \ 0001) = 1110 \ v_T \ 0001 = 1111$$

However, if we let the three instruments join *solibao* at the same time, we obtain the rhythm of *solibao*. That is, we actually hear the rhythm of the whole ensemble. Again, in terms of beat codes, we have:

$$\begin{aligned}
 0000 \ v_T (1111 \overset{\downarrow}{v_T} 1110 \ v_T 0001) &= (1111 \ v_T 1110) v_T 0001 \\
 &= 0001 v_T 0001 = 0000 \\
 0000 \ v_T (1111 \ v_T 0001 \ v_T 1110) &= (1111 \ v_T 0001) v_T 1110 \\
 &= 1110 \ v_T 1110 = 0000 \\
 0000 \ v_T (1110 \ v_T 0001 \ v_T 1111) &= (1110 \ v_T 0001) v_T 1111 \\
 &= 1111 \ v_T 1111 = 0000
 \end{aligned}$$

We can summarize the above using the following table, showing what beat code results from the join of any two beat codes (i.e. what rhythm is produced with the combination of any two rhythms):

v_T	0000	1111 (1110 v_T 0001)	1110 (1111 v_T 0001)	0001 (1111 v_T 1110)
0000	0000	1111	1110	0001
1111	1111	0000	0001	1110
1110	1110	0001	0000	1111
0001	0001	1110	1111	0000

From the table above, we can have the following observation:

1. The set C_T is closed under join, v_T , i.e., the join of any two elements of C_T is again an element of C_T .
2. $(a_1 a_2 a_3 a_4 a \ v_T \ b_1 b_2 b_3 b_4) \ v_T \ c_1 c_2 c_3 c_4$
 $= a_1 a_2 a_3 a_4 a \ v_T \ (b_1 b_2 b_3 b_4 \ v_T \ c_1 c_2 c_3 c_4)$
 since combining three rhythms taken any two at a time will create the same rhythm. This shows that v_T is associative.
3. The beat code of the *solibao* serves as the identity element.
4. Each beat code has as its inverse a beat code equal to itself. Note from the table that this inverse beat code is produced by the join of the beat codes of two other instruments. This means that we obtain the identity beat code (the beat code reflecting the rhythm of the *solibao*) only if the

three distinct instruments are played together with the *solibao*.

5. The entries in the table are symmetric with respect to the diagonal, hence, v_T is commutative.

From all these observations, we conclude that the set C_T forms an abelian group under join v_T . Therefore $\langle C_T, v_T \rangle$ is an abelian group.

Consider the Klein-4 group $V = \{e, a, b, c\}$. The one-to-one correspondence

$$\begin{array}{rcl} R : & C_T & \longrightarrow V \\ & 0000 & \longrightarrow e \\ & 0001 & \longrightarrow a \\ & 1110 & \longrightarrow b \\ & 1111 & \longrightarrow c \end{array}$$

may be used to define the elements of C_T using the elements of V . We will note that doing this will allow us to obtain the operational table of V (see Appendix A) from the operational table of C_T . This means that the operations in the two sets have the same properties.

We then say that C_T and V are structurally the same, that is, $\langle C_T, v_T \rangle$ is isomorphic to the Klein-4 group V .

B. The Pinanyowan Ensemble

For the *Pinanyowan* ensemble, on the other hand, we have the following transformed table:

Table 4b
The Transformed Beat Scheme of *Pinanyowan*

INSTRUMENT	BEAT CODE
Piyak	0 0 0 0
Tambol	1 1 1 1
Toktok	0 1 1 1
Sonob	0 1 1 1

We define the operation v_p , on the set $C_p = \{0000, 0111, 1111\}$, to be similar to v_T except that this time, it is the *piyak* that creates the basic rhythmic mode. The set has only three elements since the beat code of *toktok* is the same as that of *sonob*. However, with the *Pinanyowan*, playing the *tambol* and the *toktok* with the *piyak* creates a rhythm where the sound at the first beat is distinct from the last three beats, represented in terms of beat code by 1000. Using beat codes, we have $0111 v_p 1111 = 1000$, which is not in C_p . We then say that the operation v_p does not satisfy the closure property. Hence, v_p is not a binary operation.

Also presented in Appendix E are the transcriptions of the different gong music played, which support the observations made in the playing of the gongs.

The Algebra of the Kinship System

We will present here only those features of the Kankana-ey kinship system that we used in the mathematical analysis. A more detailed discussion of the Kankana-ey kinship system is found in Appendix F.

There are two important groups of people to which every Kankana-ey belongs. One is his personal kindred, a group of people closely related to him through both sexes, and which furnishes him with certain rights and obligations (Schusky, 1965). In Kankana-ey, the ego would usually refer to this group as his or her *sin-a-ag-i*. The other is his or her descent group or *sinpangapo*, the group of people related to him by blood, and who can actually trace their relationship to a common ancestor.

As a bilaterally structured system, the ego is surrounded by a group of relatives from both the mother and the father. In the Kankana-ey kinship, the ego recognizes as members of his or her personal kindred all relatives up to fourth degree cousins in both the father's and mother's side. All others are regarded as ordinary persons, or *ipugaw*. Some respondents, however, consider only up to second

cousins as “close” relatives. Also, part of the ego’s kindred are those belonging up to the fourth ascending and descending generations.

Bilaterality implies that a kindred is never the same for any two persons except for siblings (Schusky, 1965). This means that every child has a unique kindred, whose members are contributed by both kindreds of the parents.

On the other hand, membership to the *sinpangapo* is by descent, and this, according to Keesing (1975) is an absolute status: either a person is, or is not, a member of a particular descent group, in this case, of a particular *sinpangapo*. This means that any two *sinpangapo* are mutually exclusive. Also, there exists a finite number of distinct *sinpangapo* or groups of people who are related by blood or descendants of a common ancestor.

One interesting point: our survey showed that the Kankana-ey generally tended to favor the descent group of the father. When our respondents were asked to identify the *sinpangapo* to which they belong, *they almost instinctively responded by tracing their sinpangapo along the father’s line of descent*. It must also be noted that when they talked about their *sinpangapo*, our respondents usually would refer back to that of a founding father or a common ancestor.

This tendency to identify with the *sinpangapo* of the father does not suggest that the Kankana-ey kinship system is unilineal. In fact, all other features of this kinship system would adhere to the principle of bilaterality. For instance, when it comes to inheritance, both the father and the mother contribute to the properties that are distributed equally among their children. Some of these properties were in turn inherited by the couple from both sets of parents when they got married.

It is possible to actually deny one’s self as a member of a particular *sinpangapo*. But this is very rare, and in fact elicits a negative reaction from the community. The only case shared to us is that of a group of related individuals who completely changed their “identity” because a male ancestor committed a “shameful act” in the past.

We now present some rules governing marriage in Kankana-ey society.

San Jose (1974), in his study on the significance of epic hero Gatan in Kankana-ey culture, wrote that Kankana-ey customs “condone marriage of relatives but never of first cousins.” Indeed, marriage of first cousins, and *a priori*, of relatives closer than first cousins, is prohibited. This would mean that Kankana-ey society allows marriage of relatives beyond first cousins. However, this condoning of marriage beyond first cousins needs further qualification.

It turns out that even though Kankana-ey may allow marriage of second or third cousins, the general sentiment is to discourage such unions. The community feels that such marriages are *nagalas* (ugly) or *haan nga rebbeng* (not right). Marriage between second or third cousins used to be common among rich families, *tapno haan nga maiwaras ti daga* (so as not to distribute the land).

One important purpose of marriage for the Kankana-ey is to beget children. Children are highly valued and the acquisition of house, land or wealth becomes only secondary. It is generally considered that if a couple are childless, their marriage is incomplete. Their inability to bear children may later be a basis for separation and annulment of their marriage.

These are the features of Kankana-ey kinship system that are relevant to our mathematical analysis. To summarize, we pointed out the following:

1. Two important groups of related people every Kankana-ey belongs to. In terms of kinship, he has his unique personal kindred or *sin-a-ag-i*; in terms of descent, he belongs to one and only one *sinpangapo*.
2. Rules governing marriage.
3. The importance of children, and the identification of their kindred and descent groups. In terms of *sinpangapo* identification, the Kankana-ey tend to trace their *sinpangapo* along the father's line of descent.

A. The Use of Personal Kindred, or *Sin-a-ag-i*

The Kankana-ey society is divided into n kindreds or *sin-a-ag-i*, say $k_1, k_2, \dots, k_n$, such that every individual has a unique kindred. These kindreds are not mutually exclusive, that is, two individuals may belong to two different kindreds. For example, a mother and her daughter each has her unique kindred. However, the daughter also belongs to her mother's personal kindred and vice-versa.

Let $K = \{k_1, k_2, \dots, k_n\}$. We assume that these are all the existing kindreds in the society at a given time, such that every kindred is a result of a marriage of two individuals from two different kindreds. We give an interpretation to the set product $K \times K$. An element (k_i, k_j) of $K \times K$ represents the marriage between a man with unique kindred k_i and a woman with unique kindred k_j .

How should we interpret (k_i, k_j) ? There are two possible ways. Either we can think of it as a marriage between a man and his sister (because as siblings, they have the same kindred), or that no marriage takes place (i.e., the man or woman with kindred k_i remains single). But since marriage between siblings is prohibited, we take (k_i, k_j) to mean the second interpretation.

We now define a function $\odot: K \times K \rightarrow K$ such that for any two kindreds k_i, k_j in K , we have:

$\odot(k_i, k_j)$ = the unique kindred of their children, k_c , where k_i and k_j are the unique kindreds of the father and mother, respectively. We shall call $\odot$ **the sin-a-ag-i function**.

As mentioned earlier, we say that for any k_i in K , (k_i, k_i) would mean that no marriage takes place, so that in a sense, the elements of this kindred k_i are "preserved." Hence, $\odot(k_i, k_i) = k_i$. We note that the function $\odot$ is an onto function.

We now examine this function $\odot$ as an operation on the set K of n distinct kindreds. For any two unique kindreds k_i, k_j in K ,

$k_i \odot k_j = k_s$, the unique kindred of the children and a man with unique kindred k_i , and a woman with unique kindred k_j .

Also, for any k_i in K , we have

$$k_i \odot k_i = k_i.$$

For consistency, we shall call $\odot$ the **sin-a-ag-i operation**.

The *sin-a-ag-i* operation $\odot$ has the following properties:

1. It is well-defined because the "product" $k_i \odot k_j$ exists.
2. $k_i \odot k_j$ is unique for any two kindreds k_i and k_j .
3. $k_i \odot k_j$ is again an element of K , i.e., the set K is closed under the operation $\odot$.

Because of properties (1), (2), and (3) above, we say that the operation $\odot$ is a **binary operation** on K .

Result 1: The pair $\langle K, \odot \rangle$ is an algebraic structure.

We examine whether $\odot$ is associative, that is, we want to know whether

$$(k_i \odot k_j) \odot k_t = k_i \odot (k_j \odot k_t) \quad (\star)$$

Let us call the equation above as equation $\star$. First, we consider the expression $(k_i \odot k_j) \odot k_t$ (the left-hand side of equation $\star$). $(k_i \odot k_j)$ means that a man m_i with kindred k_i marries a woman w_j with kindred k_j . The children of this couple will have a new kindred, say k_s . A second man (son of m_i) belonging to this kindred k_s then marries a woman with kindred k_t . $(k_s \odot k_t)$ then produces a new kindred k_l . Symbolically, we have $(k_i \odot k_j) \odot k_t = k_s \odot k_t = k_l$.

Now, we consider the expression $k_i \odot (k_j \odot k_t)$, the right-hand side of the equation $\star$. A man m_i with kindred k_i marries a woman "produced" by the union of a couple with kindreds k_j and k_t . Let us say that this woman has kindred k_n . Then, $k_i \odot (k_j \odot k_t) = k_i \odot k_n$. The

outcome of this union are children belonging to a new kindred, say k_m . Symbolically, we have, $k_i \odot (k_j \odot k_t) = k_i \odot k_n = k_m$. Therefore, is equation $\star$ true? Doing it simultaneously, and based on the discussion above, we verify whether:

$$\begin{array}{rcl} & ? & \\ (k_i \odot k_j) \odot k_t & = & k_i \odot (k_j \odot k_t) \\ & ? & \\ k_s \odot k_t & = & k_i \odot k_n \\ & ? & \\ k_i & = & k_m \end{array}$$

$k_s \odot k_t \neq k_i \odot k_n$ since it is clear that $k_s \neq k_i$ and $k_t \neq k_n$ (we should keep in mind that two persons have the same kindreds if and only if they are siblings). Therefore, kindred k_i cannot be equal to kindred k_m because they are results of the combination of different pairs of kindreds. Hence, $(k_i \odot k_j) \odot k_t \neq k_i \odot (k_j \odot k_t)$, and we conclude that the *sin-a-ag-i* operation $\odot$ is not associative.

Could there be an identity element, say k_e so that for any k_i ,

$$k_i \odot k_e = k_e \odot k_i = k_i?$$

Clearly, the answer is no, for how can there be a unique kindred that would preserve the elements of all the other kindreds in the set? However what is true is that $k_i \odot k_i = k_i$ for all i , but this does not conform with our definition of an identity element in the set. The non-existence of an identity element also implies the non-existence of inverse elements.

Furthermore, we note that $\odot$ is not commutative, that is, $k_i \odot k_j \neq k_j \odot k_i$. The left-hand side of this non-equality means that we consider a man m_i (with kindred k_i) and woman w_j (with kindred k_j) whose marriage will produce children with a new kindred, say k_s . On the other hand, the right-hand side means that we also have another man m_j (with kindred k_j) marrying a woman w_i (with kindred k_i), to produce a new set of children with a new kindred, say k_t . Here, m_i and w_i , and m_j and w_j are siblings because they have the same kindreds. However, the children resulting from the marriage between man m_i and

woman w_j on the one hand, and between man m_j and woman w_i on the other hand, are in fact cousins (and not siblings). Therefore, these two sets of offsprings belong to two different kindreds, k_s and k_t .

B. The Use of *Sinpangapo*

We divide Kankana-ey society into n distinct *sinpangapo*, say $s_1, s_2, \dots, s_n$, such that each individual belongs to one and only one *sinpangapo*. As mentioned earlier, any two *sinpangapo* are mutually exclusive, that is, no individual belongs to two distinct *sinpangapo*.

Let $S = \{s_1, s_2, \dots, s_n\}$. Again we give an interpretation to the set product $S \times S$ whose elements are all the ordered pairs (s_i, s_j) , for any two s_i, s_j in S . In this case, every ordered pair (s_i, s_j) represents the marriage of a man belonging to *sinpangapo* s_i and a woman belonging to *sinpangapo* s_j .

What is meant by (s_i, s_j) ? This represents a marriage taking place within the same *sinpangapo* (or between two people who are blood related). This is perfectly alright, provided that the degree of relationship of the couple is "far enough" so as not to fall within those relationships that are considered taboo.

We define the function Δ from $S \times S$ to S as the function that assigns the children or offsprings of a marriage to a *sinpangapo*, that is,

$$\Delta(s_i, s_j) = \text{the } \textit{sinpangapo} \text{ of the children of a man with } \textit{sinpangapo} s_i \text{ and a woman with } \textit{sinpangapo} s_j.$$

We shall call Δ the ***sinpangapo* function**.

Because of the practice of identifying one's self with the *sinpangapo* of the father, we abstract this feature of the Kankana-ey kinship system so that we now have:

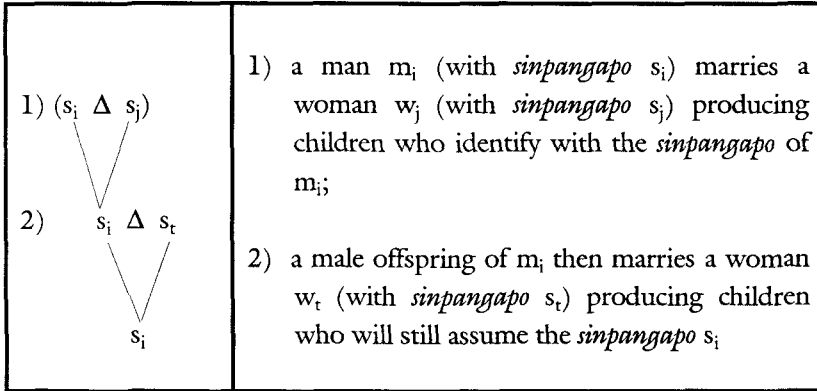
For every (s_i, s_j) in $S \times S$, $\Delta(s_i, s_j) = s_i$ (i.e., the children assume the *sinpangapo* of the father).

As an operation on the set S of all distinct *sinpangapo*, we have: for any two distinct s_i, s_j in S , $s_i \Delta s_j = s_i$. For consistency, we shall call Δ the **sinpangapo operation**.

We note the following:

1. Δ is a **binary operation on S** . This means that $s_i \Delta s_j$ exists and is unique for any two s_i, s_j in S . Furthermore, S is closed under Δ .
2. Δ is **associative**, that is, for any s_i, s_j, s_t in S , we have $(s_i \Delta s_j) \Delta s_t = s_i \Delta (s_j \Delta s_t)$ ($\star\star$)

We interpret $(s_i \Delta s_j) \Delta s_t$, the left-hand side of equation $\star\star$. First of all, $(s_i \Delta s_j)$ implies that a man m_i marries a woman w_j (who both belong to *sinpangapo* s_i and s_j , respectively), and whose union produces children who shall have the *sinpangapo* s_i , the *sinpangapo* of their father m_i . A son of m_i with *sinpangapo* s_i will then marry a woman w_t from *sinpangapo* s_t . This union in turn produces a set of siblings who would still identify with the *sinpangapo* s_i . Schematically, we have:

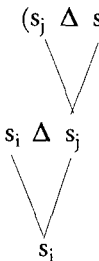


Therefore, $(s_i \Delta s_j) \Delta s_t = s_i \Delta s_t = s_i$.

We now look at the right-hand side of equation $\star\star$. How do we interpret $s_i \Delta (s_j \Delta s_t)$? The pairing $(s_j \Delta s_t)$ which represents the marriage of a man m_j and a woman w_t produces children who shall identify with the *sinpangapo* s_j . A female offspring w_j from this

marriage will marry a man m_i with *sinpangapo* s_i , producing children who will again assume *sinpangapo* s_i , the *sinpangapo* m_i , the father.

Schematically, we have:

1) $(s_j \Delta s_t)$ 2) $s_i \Delta s_j$ 	1) a man m_j (with <i>sinpangapo</i> s_j) marries a woman w_t (with <i>sinpangapo</i> s_t) producing children who identify with the <i>sinpangapo</i> s_j of m_j; 2) a female offspring w_j then marries a man m_i (with <i>sinpangapo</i> s_i) producing children who will still assume the <i>sinpangapo</i> s_i of m_i
---	--

In other words, equation $\star\star$ holds, that is,

$$(s_i \Delta s_j) \Delta s_t = s_i \Delta s_j = s_i$$

because

$$(s_i \Delta s_j) \Delta s_t = s_i \Delta s_t = s_i, \text{ while } s_i \Delta (s_j \Delta s_t) = s_i \Delta s_j = s_i.$$

Result 2: The pair $\langle S, \Delta \rangle$ is an algebraic structure. Because Δ is associative, $\langle S, \Delta \rangle$ is a *semi-group*.

Again, we note that there is no identity *sinpangapo* in the set, and that there exist no inverse elements. Also, the *sinpangapo* operation is not commutative, for $s_i \Delta s_j$ cannot be equal to $s_j \Delta s_i$ since $s_i \Delta s_j = s_i$ while $s_j \Delta s_i = s_j$ and s_i and s_j are two distinct *sinpangapo*.

In this study, we showed that it is possible to look for algebraic structures within the Kankana-ey kinship system. We did this by investigating certain features of the system that exhibit algebraic structures. Specifically, we showed that it is possible to have two

algebraic structures based on two important groupings: the personal kindred or *sin-a-ag-i*, which is based on kinship; the other is the *sinpangapo*, which is based on descent.

Using the kindreds as the elements of the set K under the kindred operation $\odot$, the pair $\langle K, \odot \rangle$ gives an algebraic structure. However, the *sin-a-ag-i* operation is neither commutative nor associative. On the other hand, using the distinct *sinpangapo* as the elements of the set S under the *sinpangapo* operation Δ , the pair $\langle S, \Delta \rangle$ is an algebraic structure. Because Δ is associative, the algebraic structure formed is a semi-group.

5

TRANSFORMING PRACTICAL KNOWLEDGE TO FORMAL KNOWLEDGE

Learning about how mathematics is imbedded in a people's culture opens a lot of opportunities, not only in deepening our mathematical and pedagogical knowledge, but also in making such knowledge more accessible and relevant to the people.

In this research, we have shown that mathematics is concretely present and operative in the indigenous culture of the Kankana-ey of Mt. Province. Specifically, we have established the existence of algebraic structures in Kankana-ey weaving patterns, gong music and kinship system.

Such a result becomes even more significant if viewed against the fact that these algebraic structures were borne out of very basic elements of their culture:

- the motifs in their fabric designs (*sopo*, *tiktiko*, *matmata*, *kulibangbang*);
- the instruments of their music (*solibao*, *takik*, *atomba*, *pingsan*); and
- their social groupings (*sin-aag-i*, *sinpangapo*)

These elements formed the underlying sets in which mathematical operations were identified. The algebraic structures thus established were the following:

Frieze groups (in the weaving patterns)

Klein-4 group (in gong music)

Semi-group (in kinship system)

What we have done is to try to understand the practical mathematical knowledge of the Kankana-ey and to render such knowledge in systematic and formal terms.

This discovery points out to the fact that many things in our culture, often taken for granted, are actually applications of abstract mathematical concepts which are worthy of study and appreciation.

Through this pioneering research, we hope to encourage other educators, especially those in Mathematics and Science, to conduct parallel studies in the other indigenous communities, not only in the Cordillera, but in the whole country. The results of this research and such parallel studies will be indispensable in developing more relevant and responsive Mathematics curricula in formal and non-formal education. These results could also serve as a basis for formulating mathematical abstractions.

In this chapter, we present sample lessons on basic mathematical concepts drawing from the indigenous knowledge of weaving, playing of gongs and kinship system, on the pedagogical assumption that students learn faster from what they already know. In these lessons, we transform these seemingly unstructured aspects of the Kankana-ey culture, which perhaps they are unaware of, into objects of formal learning. It is hoped that these lessons would show how practical community activities may be used to teach even abstract concepts in mathematics.

In terms of literate and numerate practice, these lessons seek to demonstrate how oral tradition can enter into the written tradition,

thereby promoting not only literate-numerate practice but also ensuring the preservation of the best features of our oral traditions (cf Doronila, 1996).

This chapter presents four lessons dealing with topics needed for the development of an algebraic structure, namely: the set, a binary operation on the set, properties governing the binary operation, and the algebraic structure. Each lesson is developed to include discussions on the research results on weaving, gong music and kinship system.

These lessons were developed to achieve Bloom's cognitive objective of promoting the ability to synthesize, that is the "ability to put parts together to form a new whole that was not previously present" (Gronlund, 1978). It is assumed that the target learners have gone through lessons which meet Bloom's lower cognitive objectives of promoting knowledge, comprehension, application and analysis. In other words, the learners should have gone through numeracy lessons on counting, on the four fundamental operations and on problem-solving. Equivalently, in the formal setting, these lessons may be taught to first year high school students.

LESSON 1

THE SET

Mathematical Ideas

A herd of carabaos, a bunch of bananas, the collection of the members of a descent group or *sinpang-apo*, a collection of *gangs* or gongs, and the collection of all positive even integers, have something in common. This idea of grouping objects together gives rise to the mathematical notion of a set, which we shall study.

A *set* is a collection of objects with common characteristics. The members of the set are called *elements*.

Notice that we defined set to be a collection of objects with common characteristic(s). We start with a characteristic, then collect or identify all the objects that exhibit such characteristic. The collection of these objects make up a set. Now, it may happen that not one object possesses the said characteristic. If we collect all the objects that exhibit this characteristic -- and in this case there are no such objects (we collected nothing!) -- we come up with a set that has no element. A set with no element is called an **empty set**.

A set may be described in two ways: by roster method or by rule method. In the roster method, the set is described by listing down all its elements inside the braces $\{ \}$. In the rule method, instead of listing down the elements, a descriptive phrase identifying the elements of the set is written inside the brackets. How do we symbolize the empty set? By $\{ \}$ or by $\emptyset$.

Objectives:

Content: At the end of the lesson, the student should be able to:

- a. define a set,
- b. give examples of sets

Process: The understanding of the concepts of a set will be manifested when the students will be able to:

- a. make generalizations
- b. make distinctions/contrast
- c. generate more examples, and
- d. reason/rationalize.

Strategies: Exposition, Discussion, Applications.

Materials: Overhead projector, slides, Kankana-ey fabrics, gongs or pictures.

Procedures:

Motivation

Suppose we are invited to a wedding ceremony at Demang, Sagada, Mountain Province. Let us cite what the activities are in this occasion and be ready to integrate this in the lesson we are about to study.

Who attends this occasion?

(the couple, relatives, friends, visitors, dignitaries,...)

What objects/things are usually found in such occasions?

(animals to be butchered, costumes, musical instruments, kitchen utensils, ...)

What do the old men and women wear? (Traditional costumes)

ON WEAVING

Development of the lesson

1. Let us denote by F the collection of Kankana-ey fabrics used during the wedding ceremonies.

This collection consists of the following: (Show pictures or actual objects)

woman's skirt or *tapis*

woman's waistband or *bakget*

man's loincloth or g-string or *wanes*

man's headband or *bedbed*

blanket or *ules* (used during the wedding dance ceremony)

This collection is an example of what we refer in Mathematics as set.

Using the roster method, set F may then be described as follows:

$$F = \{tapis, bakget, wanes, bedbed, ules\}$$

Using the rule method, set F may be described as

$$F = \{ \text{Kankana-ey fabrics used during wedding} \}$$

2. Now, let T be the collection of patterns seen in the *tapis*. Some members of the set are seen in the following slides. (Show slides of derived pattern # 1 - 14, 18, 25 - 28 in Table I of the research.)

3. Let us examine these patterns very closely. Note that they consist of four basic designs and their variations which are seen in Figure 4 of this research. (Show the slide of this figure).

Let D be the set consisting of these four basic designs, namely: the *tiktiko*, the *matmata*, the *sopo* and the *kulibangbang*. The set may then be described as follows:

$$D = \left\{ \begin{array}{c} \text{zigzag line} \\ \text{diamond shape} \\ \text{star shape} \\ \text{cross-hatch pattern} \end{array} \right\}$$

4. Consider first the element called *tiktiko*. Let us trace how a *tiktiko* is produced through this activity. Give each learner a cardboard strip (1 in. by $\frac{1}{8}$ in.) and Activity Sheet 1.

5. *Discuss the results of the activity.*

The half-turns about specified points, and reflections about specified lines are examples of what we call in mathematics as **isometries**. In the ordinary parlance, they are referred to as **rigid geometric motions**. As such, these motions preserve distance or length of line segments. The points and the lines are called the **fixed points** of the motions.

The set of all the half-turns about the specified points, as well as the reflections about the specified lines form the set of the motions of the given *tiktiko*.

The cardboard strip (1 in by $\frac{1}{8}$ in) is used as the **motif** for the *tiktiko*.

6. Practice

Consider designs using *matmata* and *sopo*. Distribute Activity Sheet 2 and a square cardboard (3 in by 3 in.)

- a. Determine the motif used for each design. Cut the motif out from the square cardboard.
- b. Determine the set of motions of each design, indicating the fixed points for each motion.

7. Let $G = \{ \text{women in the class who are wearing G-string} \}$. Does G contain an element? G is an example of an empty set. Can you give more examples of an empty set?

Assignment

Use Activity Sheet 2. Cut out the motif and determine the set of motions that generate the *kulibangbang* design. Indicate also the fixed points for each motion.

ON MUSIC*Development of the lesson*

There are three musical ensembles played during a wedding celebration. These are *Takik*, *Pinanyowan* and *Balangbang*. Do they form a set? Explain.

Next, we consider the musical instruments used for each ensemble. Present pictures of some of them.

1. Ask the following questions:

- a. How many sets can we form out of these instruments?
- b. Can you name some of them? [Some Possible Answers:
 1. Classify them as to whether they are gongs or not;
 2. Put in one set those which are used in one ensemble;
 3. Classify them according to the rhythm they produce.]

The following diagram illustrates the second possible answer.

*TAKIK**PINANYOWAN**BALANGBANG*

SOLIBAO
SONOB
TAKIK
ATOMBA
PINGSAN

SONOB
TAMBOL
PITYAK
TOKTOK

TOKTOK
ATOMBA
PINGSAN

Let us denote by T the set of instruments used for *Takik*. That is,

$$T = \{ \text{instruments used for Takik} \}$$

In describing our set T above, we used the rule method. We can also describe set T using the roster method as follows:

$$T = \{ \text{solibao}, \text{sonob}, \text{takik}, \text{atomba}, \text{pingsan} \}$$

Each of these instruments creates a distinct sound. This is because each is struck in a distinct way to create a unique rhythm. Let us listen carefully to the sound created by each instrument in T.

Write down your observations in the following table. [The learners are first asked to give their observations. The indicated observations express the expected answers.]

INSTRUMENT	OBSERVATION
<i>Solibao</i>	It is struck four times to set the basic rhythmic mode of four beats
<i>Takik</i>	It has the same rhythm as that of solibao.
<i>Atomba</i>	It is struck at every first three beats of the rhythmic mode.
<i>Pingsan</i>	It is struck at every fourth beat of the rhythmic mode
<i>Sonob</i>	It creates an improvisation; it follows the basic rhythmic mode.

The above observations show that each instrument creates a unique rhythm which is produced by a **beat pattern**. A **beat pattern** consists of beats repeated at regular intervals. For the *Takik* Ensemble, the basic unit is a cluster of four beats. The table below represents the basic beat pattern of each instrument by a sequence of 0s and 1s. The *sonob* does not have a beat pattern since it acts as the soloist in the ensemble.

INSTRUMENT	BASIC BEAT PATTERN REPRESENTATION
SOLIBAO	0 0 0 0
TAKIK	1 1 1 1
ATOMBA	1 1 1 0
PINGSAN	0 0 0 1

Solibao is assigned the sequence 0000 because it is played first to set the rhythmic mode of the ensemble. After the basic rhythmic mode

has been established, any other instrument can be played, at the start of any cluster. The other sequences are then obtained as follows: at the beat where an instrument is struck simultaneously with the *solibao*, we assign 1; otherwise, we assign 0 (which means that at that beat, only the *solibao* is struck relative to each instrument).

Let us denote the set of representations of the beat patterns by C_T and is described as follows:

$$C_T = \{0000, 1111, 1110, 0001\}.$$

We call each representation the **beat code** of an instrument.

Assignment

Consider the *Pinanyowan*. Listen to a recorded tape of the ensemble.

- Describe the set P of instruments used for *Pinanyowan*.
- Represent the beat patterns of each instrument (by a sequence of 0s and 1s) based on the following table:

INSTRUMENT	OBSERVATION
<i>Piyak</i>	It is struck four times to set the rhythmic mode of four beats.
<i>Toktok</i>	It is struck at every last three beats of the rhythmic mode
<i>Sonob</i>	It has the same rhythm as the toktok.
<i>Tambol</i>	It has the same rhythm as the piyak.

Note that the *piyak* is struck first to set the rhythmic mode. The other instruments are played at the start of any of the clusters which follow.

ON KINSHIP

Development of the lesson

1. Imagine it is your wedding. Who are the people you expect to show up on your wedding day? Probably they would be your parents,
 your immediate family,
 your relatives,
 your friends,
 and perhaps, some community officials and village elders.

The members of the group of people above have one thing in common -- they are related to you in some ways and they take part in the joyous celebration of your wedding. Together, they comprise what we call in mathematics a *set*.

2. If we use P to represent the set of people above, then we have

$$P = \{ \text{the people who attend your wedding} \}$$

The above representation is the rule method of describing set P . Describing set P by the roster method, we have

$$P = \{ \text{parents, family, relatives, friends, officials, elders} \}.$$

We can further let p represent parents, f family, r relatives, a friends, o officials and e elders. Thus,

$$P = \{ p, f, r, a, o, e \}.$$

3. Perhaps you are thinking, "Are not r and a sets? After all, r and a consist of people who share something in common." You are right. And p, f, o and e are sets as well. "So we have a set P composed of smaller sets?" You're right again.

3. Let $G = \{ \text{women who attend your wedding wearing G-string} \}$. Then set G contains no element. Therefore, G is an example of an empty set.

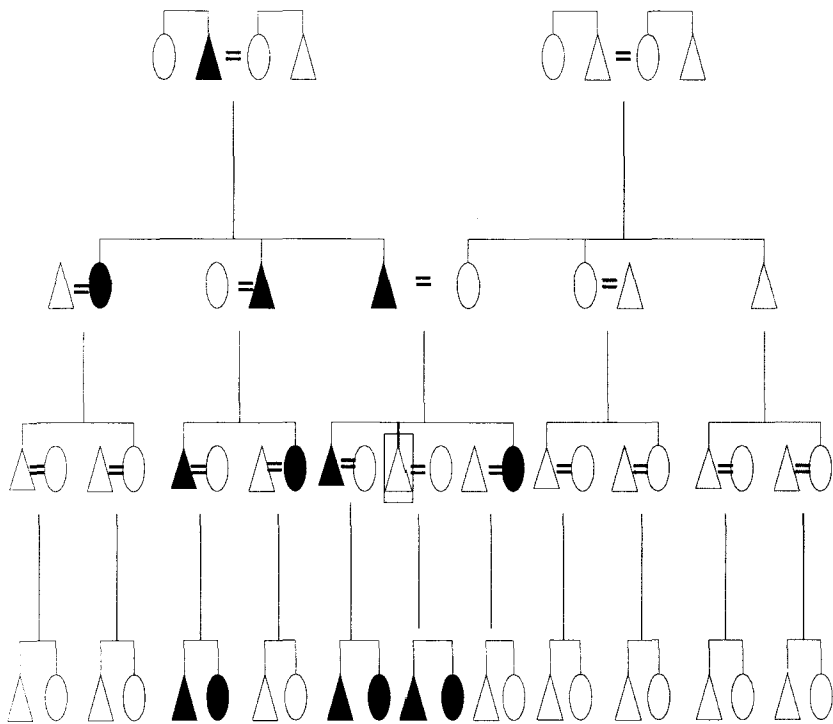
4. Can you give more examples of sets with elements? Give examples of empty sets.

5. The Kankana-ey of Mt. Province identify themselves with two groups or sets of people: the kindred and the *sinpangapo*. Say Dali-ig is a Kankana-ey. His kindred or *sin-a-ag-i* is composed of his relatives from his father and mother side, extending to his fourth degree cousins, and to his ancestors and descendants to the fourth degree. His *sinpangapo*, on the other hand, is the group of people related to him by blood or by descent from a common ancestor. And as a typical Kankana-ey, he traces his *sinpangapo* through his father side.

Diagram I on the next page shows the structure of the kinship of Dali-ig. In the diagram, the triangle represents male while the ellipse (oblong) represents female. The triangle inscribed in a rectangle represents Dali-ig (the ego). He is married and has two children. He has a brother and a sister who are married. Except his parents, siblings, wife and children, all the other people in the diagram are members of Dali-ig's *sin-a-ag-i*. The shaded triangles and circles represent the people who are members of Dali-ig's *sinpangapo*. Therefore, Dali-ig's *sin-a-ag-i* and *sinpangapo* are two different sets.

- a. Based on the diagram, identify the respective members of the *sin-a-ag-i* and *sinpangapo* of Dali-ig's father.
- b. Say you are Dali-ig's brother. Identify on the diagram the people who comprise your *sin-a-ag-i* and those who comprise your *sinpangapo*.
- c. Based on the answers you obtained from a and b, how does your *sin-a-ag-i* and *sinpangapo* differ from those of your father and your brother Dali-ig?
- d. If you are a member of, say, Yawen's kindred and Yawen is a member of your kindred, does that mean that you two have the same kindred? Does it mean that you two have the same *sinpangapo*?
- e. If Dal-lay and you have the same *sinpangapo*, does it mean that you two have the same kindred?

DIAGRAM I



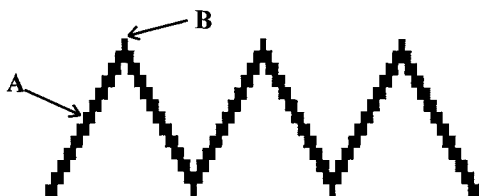
Dali-ig's Kinship Structure

ACTIVITY SHEET 1

THE *TIKTIKO*

The *tiktiko* is a zigzag-like design claimed to represent mountain foot trails as well as the zigzag mountain roads. The design will be shown to be produced by using a motif and some geometric motions. The cardboard strip serves as the motif and we identify the motions we carry out on the strip to produce the *tiktiko*.

Tack the cardboard strip on the point A of the following figure.



Ask the following questions:

1. What do you do with the strip so that you get the first A of the tiktiko?

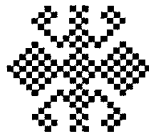
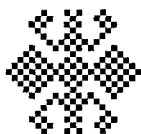
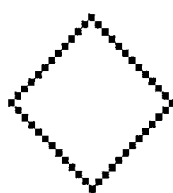
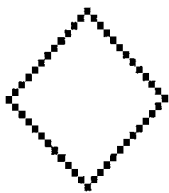
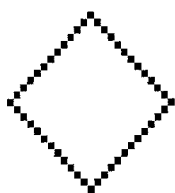
[Expected answers: First, rotate the strip about point A by 180° . In other words, make a half-turn with the strip about A. Then reflect the strip about a vertical line passing through B. Next, rotate the strip about another point by 180° . Ask where this point is.]

2. How do we get the rest of the *tiktiko*?

[Expected Answers: The rest of the tiktiko is obtained by an alternating action of half-turn of the strip about certain points and reflection of the strip about certain lines. Ask the learners to locate these points and these lines of reflection.]

3. Do the half-turns or reflections change the length and width of the cardboard strip as they act on it? Demonstrate your answer.

ACTIVITY SHEET 2



Instructions:

1. Determine the motif used for each design. Cut the motif out from the square cardboard.
2. Determine the set of motions of each design, indicating the fixed points for each motion.

LESSON 2

OPERATIONS ON THE ELEMENTS OF SETS

Mathematical Ideas:

When we multiply two numbers, we expect the product to be a number. We should be surprised if the answer was a “supot” of *pan de sal*.

It is inconceivable to have a mathematics without operations. In single terms, an operation is combining two objects in a set to produce a third object that is in the same group or set. The operations of addition, multiplication, division, subtraction are not the only operations in the realm of mathematics.

In a kinship system of a group of individuals, an operation may be defined as tracing the latest common forefather or *apo*. Thus, given two individuals, say two brothers, the latest common forefather is their father. This is an illustration of binary operation.

In the weaving process, the result of the operation will be the design. In music, the result of the interplay of the *solibao*, *takik*, *atomba*, *pingsan* and the *sonob* is the *takik* ensemble.

A *binary operation* on a set S is a rule which assigns to every two elements in the set one and only one element, which is also in S .

Objectives:

- Content: At the end of the lesson, the student should be able to:
- a. illustrate how a binary operation works in a set
 - b. discuss results of an activity
 - c. perform set operations.

Process: Generalization

Strategies: Exposition, group/individual work, simulation (e.g. making designs, actual play of *gongs*).

Materials: Recorded tapes of gong music, cardboards.

Procedures

ON WEAVING

Development of the lesson

1. **Review:** Which of the following describes a set?
 - a) { ricefield, pig, train, guava }
 - b) {the weavers in Barangay Banguitan }
 - c) {Luzon, Visayas, Mindanao }
 - d) { *tapis*, *bakget*, *wanes*, *bedbed* }
2. Distribute Worksheet 1. Explain the objective of the work and ask the learners to work in pairs. After everybody is through, ask the students to give their answers and to explain their work.
3. Discuss the results. Ask the following questions:
 - a. Does the rule assign to every two elements of set F, one and only one element from set F?
 - b. Is Δ a binary operation?
4. Distribute again Activity Sheet 1. Recall how the *tiktiko* has been generated starting from the half-turn about point A and the reflection about the vertical line which passes through B. We now denote the half-turn by **a** and the reflection by **b**.

Step by step, let us see how the pattern is built up from the cardboard strip which serves as our motif. Place **1** below the starting position, which means that it is the original position. Apply **a** on the strip. Write **a** above the obtained position. Apply **b** on the strip in position **a**. Write **ba** above the obtained position. Apply **a** again on

the strip in position ba . Write aba above the new obtained position. Repeat the procedure, that is, applying a and b alternately on the motif, until the given pattern is generated.

5. Answer the following questions.

- a. What motions are used to generate the *tiktiko* design?
[Answer: the halfturn a and the reflection b .]
- b. What did we do with the motions a and b ?
[Answer: We applied b after a , that is, a followed by b , and we write $b \odot a$.]
- c. Is $\odot$ ("followed by") a binary operation? Ask the students to explain from the definition.
- d. What does $b \odot b$ do to the cardboard strip if we place it first in position 1 ? How about $a \odot a$?
[Answer: Both $b \odot b$ and $a \odot a$ bring back the strip to position 1]

We then say that the *tiktiko* is a **pattern**, where the **generators** are a half-turn a and a reflection b . The motions that form the given pattern are sequences of a and b , applied alternately.

Explain to the class why the final pattern contains translations and reflections about other vertical lines parallel to the axis of reflection of b . For example, aba is a reflection of the original motif about a vertical line passing through point C . [Identify this point C with the class]. Also, $baba$ is a translation of the original motif. [Determine the length and direction of this translation with the class].

A **pattern** is formed by taking a motif to which are applied certain geometric transformations which preserve distance, called **isometries** (rotations, translations, reflections, glide reflections) in such a way that the aggregate of the figures so generated possesses form and symmetry.

Thus, to every pattern is associated a set of **isometries** or **rigid plane motions**, together with a binary operation. This operation is

called **functional composition**, and it is interpreted to be the action "followed by" of the plane motions.

6. Practice

- a. Using Activity Sheet 2, determine a set of generators of the first pattern. In other words, what are the motions which when applied on the motif, one after the other, produces the pattern?
- b. Label each position of the motif, as we did in Activity Sheet 1.
- c. What operation is performed on the elements of the set of motions which produce the pattern? Is it a binary operation?

Assignment

Using Activity Sheet 2, determine a set of generators of the second pattern. Label each position of the motif. What operation is defined on the set of motions which produce the pattern?

ON MUSIC

Development of the lesson

1. Discuss briefly the assignment given in lesson 1 with the learners.

2. Now let us go back to our set $E = \{Takik, Balangbang, Pinanyowan\}$. Let us recall the number of instruments for each ensemble. *Balangbang* has 3; *Pinanyowan*, 4; and *Takik*, 5.

Then, we define a rule that will assign to a pair of elements of E , that which uses the lesser or the same number of instruments. Let us denote it by "#". For example,

$$Takik \# Balangbang = Balangbang$$

This concept actually tells us the rule on the elements of the set. Who sets this rule? You do, but this rule needs to be well-defined, which means there is always an answer and the answer is in the set.

The following table presents the results of applying the defined rule.

#	<i>Balangbang</i>	<i>Pinanyowan</i>	<i>Takik</i>
<i>Balangbang</i>	<i>Balangbang</i>	<i>Balangbang</i>	<i>Balangbang</i>
<i>Pinanyowan</i>	<i>Balangbang</i>	<i>Pinanyowan</i>	<i>Pinanyowan</i>
<i>Takik</i>	<i>Balangbang</i>	<i>Pinanyowan</i>	<i>Takik</i>

3. Ask the following questions:

- Does the rule # assign to every two elements of E one and only one element of set E?
- Is # a binary operation?

4. Consider the set $C_T = \{0000, 1111, 1110, 0001\}$ of representations of the beat patterns for the instruments of *Takik*. Let us call such representations as **beat codes**. What does the simultaneous playing of two instruments do to their beat patterns? Let us listen to a recorded playing of the *Takik* ensemble.

This research presents a rule which assigns to every pair of the beat codes in C_T , another beat code. We call this rule “join” and is denoted by v_T . The results are presented in the following table:

v_T	0000	1111	1110	0001
0000	0000	1111	1110	0001
1111	1111	0000	0001	1110
1110	1110	0001	0000	1111
0001	0001	1110	1111	0000

Assignment Answer the following questions:

- Interpret the table of operation, v_T .
- Is v_T a binary operation? Explain from the definition.
- Copy the table on a piece of cardboard.

ON KINSHIP

Development of the lesson

1. The Binary Operation

Let us return to our wedding ceremony at Demang, Sagada, Mr. Province. We can classify or group the people attending there into sets, like in the following manner:

E = the set of elders (age 50 above)

M = the set of middle-aged adults (age between 30 and 50)

Y = the set of young adults (age 20 to 30)

T = the set of teenagers (age 13 to 19)

C = the set of children (age 12 and below)

In doing so, we see that our big set W (the set of all people attending the wedding) is composed of small sets. We have a set of sets. In symbols, we have

$$W = \{ E, M, Y, T, C \}$$

Exercise 1. What other small sets can you define that may be found in the wedding?

Let us define a rule or operation “plus” on W such that for any two elements a and b , the result to “ a plus b ” is the set containing elements that belong to set a and to set b . For example, E plus M will give us a new set containing all elders and middle-aged adults. Y plus C results in a set containing all young adults and children.

Exercise 2. What is the result of the following?

a. T plus M

b. M plus Y

c. C plus E

Let us define another rule or operation Φ on W such that $a \Phi b$ will give us the set whose members are older. Thus $E \Phi M = E$ because the members of E (age 50 and above) are older than the members of M (age between 30 and 50). Similarly,

$$Y \Phi C = Y.$$

Exercise 3. What is the result of the following?

a. $T \Phi M$

b. $M \Phi Y$

c. $C \Phi E$

Exercise 4. Get the Φ on all possible pairs (a, b) , where $a, b \in W$.

Exercise 5. So far we have seen two rules or operations on W : plus and Φ . Which among the two is a binary operation on W ?

Now let us construct two interesting sets of sets.

2. The *Sin-a-ag-i* or Kindreds

Our first set has kindreds or *sin-a-ag-i* as its elements. If K represents the set of all kindreds in your community in a given time and there are n kindreds, then we have

$$K = \{ k_1, k_2, k_3, \dots, k_n \}.$$

We are now in a position to define an operation on our set of kindreds.

Let us take any two kindreds k_i and k_j . We define the operation $\odot$ as that which assigns the kindred to the offspring of a man from k_i and a woman from k_j . In symbols,

$$k_i \odot k_j = k_t, \text{ where } k_t \text{ is the kindred of the offspring of a man with kindred } k_i \text{ and a woman with kindred } k_j.$$

We point out that k_i is the kindred of the man while k_j is the kindred of the woman. Thus

$k_1 \odot k_2$ gives the kindred of the offspring of a man with kindred k_1 and a woman with kindred k_2 , while

$k_2 \odot k_1$ gives the kindred of the offspring of a man with kindred k_2 and a woman with kindred k_1 .

- a. What do the following mean?
 - i. $k_5 \odot k_8$
 - ii. $k_{10} \odot k_4$
 - iii. $k_m \odot k_n$
 - iv. $k_n \odot k_m$
- b. Say you are already married and has kindred k_3 , while your mate has kindred k_7 . Describe the kindred of your children.
 - i. If you are male, the kindred of your children is $_\odot_\$.
 - ii. If you are female, the kindred of your children is $_\odot_\$.

3. The *Sinpangapo*

Our second set consists of the *sinpangapo*. Let us say that in a given community there are n *sinpangapo*. That is, if S is the set of all *sinpangapo* in the community, then

$$S = \{ s_1, s_2, s_3, \dots, s_n \}.$$

Let us take any two *sinpangapo* s_i and s_j . Let us define a rule or operation on the set S , denoted by Δ , where

$$s_i \Delta s_j = \text{the } \textit{sinpangapo} \text{ of the children of a man with } \textit{sinpangapo} \text{ } s_i \text{ and a woman with } \textit{sinpangapo} \text{ } s_j.$$

For example: $s_1 \Delta s_2$ gives the *sinpangapo* of the children of a man with *sinpangapo* s_1 and a woman of *sinpangapo* s_2 .

The Kankana-ey practice is to identify one's self with the *sinpangapo* of the father. So for the Kankana-ey in particular,

$$s_i \Delta s_j = s_i.$$

Hence, $s_1 \Delta s_2 = s_1$.

- a. Answer the following and interpret the results.
 - i. $s_1 \Delta s_5 =$
 - iii. $s_m \Delta s_n =$
 - ii. $s_3 \Delta s_9 =$
 - iv. $s_n \Delta s_m =$
- b. Say you are a Kankana-ey, married and has *sinpangapo* s_3 , while your mate has *sinpangapo* s_7 . Describe the *sinpangapo* of your children.

- ii. If you are male, the *sinpangapo* of your children is
 $_ \Delta _ = _$.
- ii. If you are female, the *sinpangapo* of your children is
 $_ \Delta _ = _$.
- c. We claim that $\odot$ and Δ are binary operations. Explain why.

WORKSHEET 1

Consider the set $F = \{tapis, bakget, wanes, bedbed\}$. In the market, I found out that the fabrics in set F cost as follows:

TAPIS - P350.00

BAKGET - 150.00

WANES - 500.00

BEDBED - 100.00

On F, define a rule Δ such that for two elements *Tapis*, *Bakget* in F

Tapis Δ *Bakget* = the fabric which costs more.

Thus, we have the following:

TAPIS Δ *BAKGET* = *TAPIS*

TAPIS Δ *WANES* = *WANES*

Complete the following table following the given rule.

Δ	TAPIS	BAKGET	WANES	BEDBED
TAPIS		TAPIS	WANES	
BAKGET				
WANES				
BEDBED				

LESSON 3

PROPERTIES OF AN OPERATION
ON THE ELEMENTS OF A SET

Mathematical Ideas

In this lesson we shall look at the rules that govern operations on sets and how the elements of the sets may be generated from these properties.

Associative Property

A binary operation $\bullet$ on a set S is said to be *associative* if for every three elements x, y , and z in S ,

$$(x \bullet y) \bullet z = x \bullet (y \bullet z).$$

Identity Property

A set S has an *identity element* e under a given operation $\bullet$ if

$$x \bullet e = e \bullet x = x, \quad \text{for every element } x \text{ in } S.$$

We then say that the operation satisfies the identity property.

Inverse Property

A set S , with a binary operation $\bullet$, is said to satisfy the *inverse property* if it has an identity element, say e , such that for every element x in S , there is an element x^{-1} in S , with

$$x \bullet x^{-1} = e = x^{-1} \bullet x \quad \text{for every element } x \text{ in } S.$$

The element x^{-1} is called the inverse of x .

Commutative Property

A binary operation $\bullet$ on a set S is said to be *commutative* if for every two elements x and y in S ,

$$x \bullet y = y \bullet x$$

In the weaving pattern or in the *takik* ensemble, the end product of the beautiful design or musical regularity will not materialize if the process or operation has not been governed by a well-defined set of properties.

Objectives:

Content: At the end of the lesson, the student should be able to:

- a. enumerate some properties of some set operations
- b. discuss the properties of these set operations.
- c. give examples using sets identified in the wedding ceremony activity.

Process: Generalization, Application, Proving

Strategies: Exposition, Discussion, Application

Materials: slides, overhead projectors, Kankana-ey fabrics

Procedure:

On Weaving

Development of the lesson

1. Discuss briefly the assignment of lesson 2 with the learners.

2. Now, let us transform our classroom into a market. I sell Kankana-ey fabrics, specifically *tapis*, *bakget*, *wanes* and *bedbed*. Let us recall the prices for each item which we used in Worksheet 1. [Have samples with price tags on the table.]

Distribute Worksheet 1. Review the operation on set $F = \{\textit{tapis}, \textit{bakget}, \textit{wanes}, \textit{bedbed}\}$, a rule which assigns to any two fabrics in F , the fabric which costs more. For example, for the items, a *wanes* and a *tapis*, the one which costs more is the *wanes*.

a. Ask one learner to pose as a buyer who would like to buy three items. Ask him to look at the price tags and determine the fabric which costs more among the three chosen fabrics.

Recall that the defined operation is binary, that is, we apply it on two fabrics at a time. This time, let us try to do that with the three chosen items. Let us assume them to be a *tapis*, a *bakget*, and a *bedbed*. Illustrate by applying the given operation on two fabrics at a time, say a *tapis* and a *bakget*, which results to a *tapis*; then apply the operation on this result (a *tapis*) and a *bedbed*. We still get a *tapis*. In mathematical symbols, what we did may be written as follows:

$$(Tapis \Delta Bakget) \Delta Bedbed = Tapis \Delta Bedbed = Tapis$$

Suppose, we apply the operation on a *tapis* and the result of the operation on a *bakget* and a *bedbed*, which is a *bakget*. In other words, we are applying the operation on a *tapis* and a *bakget*; and we get a *tapis*. In mathematical symbols, what we did is as follows:

$$Tapis \Delta (Bakget \Delta Bedbed) = Tapis \Delta Bakget = Tapis.$$

Thus, we have

$$Tapis \Delta (Bakget \Delta Bedbed) = (Tapis \Delta Bakget) \Delta Bedbed = Tapis$$

As we consider three fabrics, we can check that we get the same result, in whatever way we put together two fabrics at a time, provided we do not change the order of the operands. We then say that the operation Δ is associative.

b. Let us go back to the operation table in Worksheet 1. What element in F satisfies all of the following?

$$Tapis \Delta \underline{\hspace{1cm}} = Tapis = \underline{\hspace{1cm}} \Delta Tapis$$

$$Bakget \Delta \underline{\hspace{1cm}} = Bakget = \underline{\hspace{1cm}} \Delta Bakget$$

$$Wanes \Delta \underline{\hspace{1cm}} = Wanes = \underline{\hspace{1cm}} \Delta Wanes$$

This element is what we call the *identity element* of set F under the operation Δ . It is an element which leaves every element in the set unchanged under the given operation.

Ask the following questions:

- [Expected Answer: “No motion” or “as you were” element. Denote this identity element by **i**. Illustrate on the figure in Activity Sheet 1 that for the half-turn **a** and the reflection **b**,

Explain that “no motion” or as you were is a rigid motion, that is applying it on a motif will not change the size of the motif.

- c. Let us go back to the operation in Worksheet 1, which is taking the fabric which costs more of two fabrics. Consider an element, say *tapis*. Ask the learners if there is an element in the set which, when paired with *tapis* through the given operation will result to the identity element, *bedbed*. [Answer: None]

Tapis Δ _____ = *Bedbed* *Wanes* Δ _____ = *Bedbed*

$$\text{Bakget} \Delta \text{ } = \text{Bedbed} \quad \text{Bakget} \Delta \text{ } = \text{Bedbed}$$

Discuss the results.

Each of the obtained elements, if any, is called the inverse to the given element. In a sense, the operation combines an element and its inverse element to come up with the identity element, *bedbed*.

d. Let us go back to our stall of Kankana-ey fabrics in the market. Take again any two of these fabrics (say, a *tapis* and a *bakget*).

What results from the application of the operation Δ (the fabric which costs more) on the two fabrics? Note that we get the same result (*tapis*) with the two expressions whether we have $\text{Tapis} \Delta \text{Bakget}$ or $\text{Bakget} \Delta \text{Tapis}$. Mathematically, we write

$$\text{Tapis} \Delta \text{Bakget} = \text{Bakget} \Delta \text{Tapis} = \text{Tapis}$$

Is this true for any two elements in our set F of Kankana-ey fabrics?

Based on the given definition, is Δ commutative?

3. Practice:

Consider the set of motions which generate the *tiktiko* design in Activity Sheet 1. Recall with the class that the identity element for this set is the “no motion” or “as you were”. In other words, it keeps the motif in the original position.

Apply the half-turn **a**, on the motif which is in the original position. What should be applied next on the motif to bring it back to its original position? [Answer: **a**] Do this also for the reflection **b**. [We should also get **b** as its inverse element]. Mathematically, we then write

$$\mathbf{a} \odot \mathbf{a} = \mathbf{i} \quad \text{and} \quad \mathbf{b} \odot \mathbf{b} = \mathbf{I}$$

Note that the inverse of a half-turn is itself and that of a reflection is also itself. Verify this using the other half-turns and reflections in the set of motions which generate the *tiktiko* design.

Assignment

- a. Determine the inverse elements of each motion to generate the patterns in Activity Sheet 2.
- b. Is the operation Θ (followed by) defined on the set of motions to produce the *tiktiko* in Activity Sheet 1 commutative?

ON MUSIC

Development of the lesson

1. Consider again the set $E = \{\text{Balangbang}, \text{Pinanyowan}, \text{Takik}\}$ and the operation $\#$. If this operation is applied on the three elements of E , what would be the result? In terms of the number of instruments used for each ensemble, how would you describe the result?

Since the defined operation is binary, we apply it on two ensembles at a time. Let us try to do that with the three elements *Balangbang*, *Pinanyowan* and *Takik*, in that order. We then write

$$(\text{Balangbang} \# \text{Pinanyowan}) \# \text{Takik} = ?$$

while

$$\text{Balangbang} \# (\text{Pinanyowan} \# \text{Takik}) = ?$$

Are the two results equal? Compare the two given expressions on the left, state their similarities and differences.

Note that the result of applying the operation in whatever way we put together two elements at a time is the same, provided we do not change the order of the operands. In like manner, other possibilities of

ordering the operands can be considered as an exercise to show that the operation is **associative**.

2. Carry out this activity in class with half of them doing part a and the other half, doing part b.

Answer the following:

$$\begin{array}{ll} \text{a. } 0 + _ = 0 = _ + 0 & 3 + _ = 3 = _ + 3 \\ 1 + _ = 1 = _ + 1 & 4 + _ = 4 = _ + 4 \\ 2 + _ = 2 = _ + 2 & 5 + _ = 5 = _ + 5 \end{array}$$

Note that the operation is the usual addition.

$$\text{b. } \textit{Balangbang} \# \textit{Balangbang} = _ = \textit{Balangbang} \# \textit{Balangbang}$$

$$\textit{Balangbang} \# \textit{Pinanyowan} = _ = \textit{Pinanyowan} \# \textit{Balangbang}$$

$$\textit{Balangbang} \# \textit{Takik} = _ = \textit{Takik} \# \textit{Balangbang}$$

- c. In problem a above, how many possible answers do you have for the blanks?
- d. What happens to your answers to the other numbers when the operation $+$ is applied?
- e. In part b above, what is (are) the result(s)?
- f. Based on the definition of an identity, what is the **identity element** for $+$? For $\#$?

3. We have already established a unique identity for our set E, the *Balangbang*. Group the learners in pairs and carry out the following activity.

		<u>result</u>	
i. <i>Balangbang</i>	# _____ =	<i>Balangbang</i>	The unique identity for # on E
ii. <i>Pinanyowan</i>	# _____ =	<i>Balangbang</i>	
iii. <i>Takik</i>	# _____ =	<i>Balangbang</i>	

The above activity asks us to find an element of set E, which when paired with any element of E produces the unique identity. Note, however, that only the first has an answer. Here, we say that only the element *Balangbang* has an **inverse**, which is itself (the element to be filled on the blank). The other two elements do not have inverses.

4. Using the same set E and the same operation #, state whether each of the following statements is true or false. **Group the learners in pairs and encourage discussion.**

<i>Balangbang</i> # <i>Takik</i> = <i>Takik</i> # <i>Balangbang</i>	_____
<i>Balangbang</i> # <i>Pinanyowan</i> = <i>Pinanyowan</i> # <i>Balangbang</i>	_____
<i>Pinanyowan</i> # <i>Takik</i> = <i>Takik</i> # <i>Pinanyowan</i>	_____

Is the operation # **commutative**, based on the definition of commutativity of an operation? Explain your answer.

Assignment:

1. After identifying the unique identity for v_T on the set $C_T = \{0000, 1111, 1110, 0001\}$, determine the inverse of each of the elements by filling in the blanks with the correct answer in the table below.

		<u>result</u>
1. 0000 v_T	_____ =	identity
2. 1111 v_T	_____ =	identity
3. 1110 v_T	_____ =	identity
4. 0001 v_T	_____ =	identity

2. Is v_T associative on C_T ? (There are 24 possible combinations of three elements from C_T .)

3. Is v_T commutative on C_T ?

ON KINSHIP

Development of the lesson

1. Review:

In our last lesson, we constructed the set S of all *sinpangapo* and we defined an operation Δ on it. We also constructed the set K of all kindreds and we defined a rule $\odot$ on it. In this lesson, we will look at some properties or axioms governing a binary operation defined on a set. We will then examine our set S with Δ and set K with $\odot$ relative to these properties.

Consider again our set $W = \{ E, M, Y, T, C \}$ under the operation Φ . If we take any three elements of W , say a, b , and c , how does the result of $(a \Phi b) \Phi c$ compare with $a \Phi (b \Phi c)$? Let us get some examples and find out.

$(E \Phi M) \Phi Y = \underline{\hspace{2cm}}$	$E \Phi (M \Phi Y) = \underline{\hspace{2cm}}$
$(E \Phi Y) \Phi T = \underline{\hspace{2cm}}$	$E \Phi (Y \Phi T) = \underline{\hspace{2cm}}$
$(C \Phi T) \Phi M = \underline{\hspace{2cm}}$	$C \Phi (T \Phi M) = \underline{\hspace{2cm}}$
$(T \Phi Y) \Phi C = \underline{\hspace{2cm}}$	$T \Phi (Y \Phi C) = \underline{\hspace{2cm}}$

Write down your observation: _____

Can you say that your observation is true for any three elements of set W ?

Let us go back to our set S of all *sinpangapo* and the operation Δ we defined on it. To review, $S = \{ s_1, s_2, s_3, \dots, s_n \}$, the set of

sinpangapo of the Kankana-ey society at a given time, while $(s_i \Delta s_j)$ gives us the *sinpangapo* of the offsprings of a man from *sinpangapo* s_i and a woman from *sinpangapo* s_j . We know of course that $s_i \Delta s_j = s_i$, that is, the offsprings usually identify themselves with the father's *sinpangapo*.

Our interest is: Is (S, Δ) associative? In effect, we want to find out whether $(s_m \Delta s_n) \Delta s_k = s_m \Delta (s_n \Delta s_k)$, for any three *sinpangapo* s_m, s_n, s_k . We take any three elements in S and do some operations, just like in the table below. Supply the missing information.

Equal? (Y/N)			
$(s_1 \Delta s_2) \Delta s_3 =$	___	$s_1 \Delta (s_2 \Delta s_3) =$	___
$(s_4 \Delta s_6) \Delta s_2 =$	___	$s_4 \Delta (s_6 \Delta s_2) =$	___
$(s_7 \Delta s_5) \Delta s_{11} =$	___	$s_7 \Delta (s_5 \Delta s_{11}) =$	___
$(s_{10} \Delta s_1) \Delta s_8 =$	___	$s_{10} \Delta (s_1 \Delta s_8) =$	___

State your observation here: _____

Conclusion: The operation Δ defined on the set S of *sinpangapo* of the Kankana-ey society at a given time, is associative.

Question: Is the operation $\odot$ on our set K of all kindreds of the Kankana-ey society in a given time associative?

3. Consider again our set W under the operation Φ . Take any one of its elements. What will happen if we combine that element with the youngest element C under Φ ? To see it, just complete the table below.

other element Φ C	result	C Φ other element	result
E Φ C		C Φ E	
M Φ C		C Φ M	
Y Φ C		C Φ Y	
T Φ C		C Φ T	
C Φ C		C Φ C	

Write down your observation: _____

The *identity element* in our set W under Φ is the set of children.

- a. Does our set K under $\odot$ have an identity element? Why or why not?
- b. Does our set S under Δ have an identity element? Why or why not?

What is the identity element of W under Φ ? [Answer: The element C (the set of female children)]. Notice that $C \Phi C = C$. We therefore say that the inverse of the element C under Φ is itself, C . What about the other elements, what are their respective inverses?

4. Let us consider the element E (the set of all elders). Under the operation Φ , we cannot find an element, say a , in W such that $E \Phi a = E$. We therefore say that the element E has no inverse.

- a. Determine whether or not the other elements of W have their respective inverses.
- b. In the sample sets you gave, what are the inverses of the elements?

Our sets S and K with operations Δ and $\odot$, respectively, do not have the identity element. Since the inverse depends on the existence of the identity element, we cannot speak of inverses of the elements of S or K .

5. a. Get any two elements of $\langle W, \Phi \rangle$, say a and b . Is $(a \Phi b)$ equal to $(b \Phi a)$? To help you decide on it, complete the table below.

a Φ b	Are the Results Equal?	b Φ a
E Φ M = <u> </u>	Yes/No	M Φ E = <u> </u>
C Φ T = <u> </u>	Yes/No	T Φ C = <u> </u>
Y Φ E = <u> </u>	Yes/No	E Φ Y = <u> </u>
M Φ T = <u> </u>	Yes/No	T Φ M = <u> </u>

- i. List all the other possible pairs of elements on set W , as in the table above.
 - ii. What is your observation? _____
- b. This time, using $\langle S, \Delta \rangle$, repeat the same steps you did in a above.
- c. Using $\langle K, \odot \rangle$, repeat the same steps you did in b above.

Is $\langle W, \Phi \rangle$ commutative? What about $\langle K, \odot \rangle$ and $\langle S, \Delta \rangle$?

LESSON 4

ALGEBRAIC STRUCTURES

Mathematical Ideas:

The word "structure" has been used to refer to almost anything which builds up from basic elements into a unified entity. The idea may be applied to a social structure, political structure, or simply, the structure of a building.

A social structure may be associated with its community customs, traditions, beliefs or community relations. In engineering and architecture, 'structure' retains its original meaning.

However, the concept of a mathematical structure should be distinguished from the structures mentioned above. To clearly understand a mathematical structure, or more specifically an algebraic structure, we shall integrate the previous lessons. Thus, the concept of a set, the operations involving sets, and their properties will now be integrated into a unified structure.

The set A , together with a defined binary operation $*$, satisfying certain properties is called an algebraic structure. In symbols, we write

$\langle A, * \rangle$ is an algebraic structure.

Specifically:

- If the defined operation on the set satisfies the associative property, the algebraic structure is called a *semi-group*.
- If the defined operation on the set satisfies the associative and identity properties, the algebraic structure is a *monoid*.
- If the defined operation satisfies the associative, identity and inverse properties, the algebraic structure is a *group*.
- If in addition to the given properties, the defined operation on the set satisfies the commutative property, we call the set with the operation a *commutative algebraic structure* (semi-group or monoid or group). A commutative group is also called an *abelian group*.

Objectives:

Content: At the end of the lesson, the student should be able to:

- a. define an algebraic structure
- b. identify the algebraic structures drawn from three features of Kankana-ey society.

Process: Make generalizations, distinctions.

Strategies: Exposition, Discussion, Group Work.

Materials: Overhead projector, slides, samples of *tapis*, objects which can be substituted for musical instruments in place of *gongs*.

Procedures:

1. Review.
 - a. Have the learners recall the sets where we defined a binary operation.
 - b. Call on the learners to enumerate the properties of each of the defined binary operations.
2. Development of the Lesson.
 - a. Distribute Activity Sheets 3, 4 and 5. Fill in the blanks as a class, making clarifications, if necessary.
 - b. Discuss the results of the activity.
 - c. Complete the statements at the bottom of each Activity Sheet. Discuss the results.
 - d. Discuss some implications of the existence of algebraic structures in gong music and the Kankana-ey kinship system.

The group of isometries (rigid motions) which forms a pattern which repeats in one direction is called a **frieze group**. The pattern formed is called a **frieze pattern** or **strip pattern**.

It is perhaps surprising to learn that once the motif has been selected, the number of ways of developing a pattern from it by

combinations of rigid motions is strictly limited. There are exactly seven distinct frieze patterns.

The research “The Algebra of Kankana-ey Weaving Patterns, Gong Music and Kinship System” has established that the Kankana-ey weaving patterns can be classified into four of these seven frieze patterns.

ACTIVITY SHEET 3 (On Weaving Patterns)

INSTRUCTIONS: Write the examples of sets and the corresponding operations on the indicated blanks.

SET: $F =$ _____; $B =$ _____;
 $C =$ _____

OPERATION: $\Delta =$ _____; $\Theta =$ _____;
 $\odot =$ _____

PROPERTIES OF THE OPERATION:

Check out the property(ies) of the operation, which we defined above.

PROPERTY	Δ	Θ	$\odot$
Associativity			
Identity			
Inverse			
Commutativity			

CONCLUSIONS: 1) $\langle A, \Delta \rangle$ is a _____.

2) $\langle B, \Theta \rangle$ is a _____.

3) $\langle C, \odot \rangle$ is a _____.

ACTIVITY SHEET 4
(On Gong Music)

INSTRUCTIONS: Write the examples of sets and the corresponding operations on the indicated blanks.

SET: $E =$ _____; $C_T =$ _____;
 $C_P =$ _____

OPERATION: $\# =$ _____; $v_t =$ _____;
 $v_p =$ _____

PROPERTIES OF THE OPERATION:

Check out the property(ies) of the operation, which we defined above.

PROPERTY	$\#$	v_t	v_p
Associativity			
Identity			
Inverse			
Commutativity			

CONCLUSIONS: 1) $\langle E, \# \rangle$ is a _____.

2) $\langle C_T, v_t \rangle$ is a _____.

3) $\langle C_P, v_p \rangle$ is a _____.

ACTIVITY SHEET 5

(On Kinship System)

INSTRUCTIONS: Write the examples of sets and the corresponding operations on the indicated blanks.

SET: $W =$ _____; $K =$ _____;
 $S =$ _____

OPERATION: $\Phi =$ _____; $\odot =$ _____;
 $\Delta =$ _____

PROPERTIES OF THE OPERATION:

Check out the property(ies) of the operation, which we defined above.

PROPERTY	Φ	$\odot$	Δ
Associativity			
Identity			
Inverse			
Commutativity			

CONCLUSIONS: 1) $\langle W, \Phi \rangle$ is a _____.

2) $\langle K, \odot \rangle$ is a _____.

3) $\langle S, \Delta \rangle$ is a _____.

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APPENDIX A

BASIC DEFINITIONS AND PROPERTIES

For a better comprehension of this report, definitions of some mathematical terms and concepts are presented here.

A SET is a well defined collection or aggregate of objects of any kind. A non-empty set is then a set with at least one member or element. Numbers are mathematical objects and thus can be considered elements of a set. Sets that we usually use are the set of integers ($\mathbb{Z}$), the set of rational numbers ($\mathbb{Q}$), the set of real numbers ($\mathbb{R}$).

A BINARY OPERATION $*$ on any set S is a rule which assigns to each ordered pair of elements of S a unique element of the same set. Thus, a binary operation may be considered as a function $*$ from $S \times S$ to S , usually written as $*$: $S \times S \rightarrow S$. It is important to identify here the set S since it can happen that the same rule is not a binary operation with respect to another set, say, M .

This definition of a binary operation requires that: (1) the third element assigned to an ordered pair is unique; (2) the third element is well defined; and (3) $*$ has the CLOSURE PROPERTY : that is, given two elements x and y of S , $x*y$, which is equal to a third element z , is again an element of the same set S .

An example of a binary operation on the set $\mathbb{Z}$ is the operation "+". If we take two elements 2 and 3, $2+3 = 5$ is also in $\mathbb{Z}$.

AN ALGEBRAIC STRUCTURE consists of a non-empty set S and at least one binary operation $*$ on S , denoted by $\langle S, * \rangle$. Two binary operations on two sets S and G may produce the same type of algebraic structure. The condition is simple: the elements of both sets under their respective binary operations must satisfy the same set of properties. These properties are given below.

Let $*$ be a binary operation on a set S .

Property 1: ASSOCIATIVE PROPERTY

For any elements x, y and z of S , $(x*y)*z = x*(y*z)$.

Property 2: EXISTENCE OF AN IDENTITY ELEMENT

There exists a unique element e (called the unique identity) of S such that $x*e = e*x = x$, where x is any element of S .

Property 3: EXISTENCE OF INVERSES

For every element x of S , there exists a unique element x^{-1} (called the unique inverse of x) also in S such that

$$x*x^{-1} = x^{-1}*x = e.$$

Property 4: COMMUTATIVE PROPERTY

For any elements x and y of S , $x*y = y*x$.

A SEMI GROUP is an algebraic structure which is one-operational and which satisfies Property 1.

A GROUP is also an algebraic structure which is one-operational, satisfying Properties 1 to 3. If, in addition, the group satisfies Property 4, then we have an ABELIAN GROUP.

One example of an abelian group is the Klein-4 (or V) group. This is composed of the set $S = \{e, a, b, c\}$ of four elements and a binary operation $*$ which when applied to S gives the following table:

$*$	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e

The table gives us the following information:

$$a*a = b*b = c*c = e*e = e$$

and

$$a*b = b*a = c; \quad a*c = c*a = b; \quad b*c = c*b = a.$$

With $*$ defined above, it is easy to check that $\langle S, * \rangle$ satisfies all properties mentioned earlier. This shows that S under the binary operation $*$ is an abelian group.

A ONE-TO-ONE CORRESPONDENCE between two sets S and G is a pairing between their elements in such a way that each element of S is paired with exactly one element of G and, similarly, each element of G is paired with exactly one element of S .

Given a set G and a binary operation $\otimes$, another set S and a binary operation $*$. We say that G and S are structurally the same if (1) they can be put into a one-to-one correspondence; (2) the same set of properties are satisfied by both structures; and (3) if we rename every element of G using the the elements of S according to the rule of correspondence, the operational table of S is obtained.

If this is the case, we say that G is ISOMORPHIC to S , or in symbol, $G \approx S$.

APPENDIX B

BASIC CONCEPTS ON FRIEZE GROUPS

For the weaving component, the following mathematical terms below have to be clarified.

A TRANSFORMATION on the plane is a one-to-one correspondence from the set of points on the plane onto itself, that is, it is a rule that associates to every point on the plane exactly one point also on the plane. The association can also be described as a pairing of point (P, P^1) . This means that a transformation t takes the point P to point P^1 , i.e., $t(P) = P^1$, so that the pair has a first component P and a second component P^1 . A point P is called an INVARIANT POINT of a transformation if P^1 coincides with P . In other words, the transformation leaves P fixed.

In our discussion, points on the plane will be denoted by capital letters, and lines will be denoted by small letters. Transformations will be denoted by Greek letters. An ISOMETRY is a transformation which preserves length so that if (P, P^1) and (Q, Q^1) are two pairs of corresponding points, then the line segments PQ and P^1Q^1 are congruent. The following figure illustrates this:

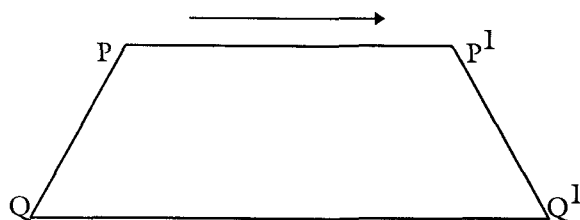


Figure 1

Rotations, translations and reflections are examples of isometries. A **rotation** of the plane about a point P is an isometry which fixes P , i.e. P is the only invariant point. The symbol, s_P , denotes a rotation about point P . A rotation through a 180° -degree angle is called a **half turn**. The direction of rotation is not important in a halfturn. We will consider only half turns in our discussion so we illustrate a halfturn about a point P in Figure 2.

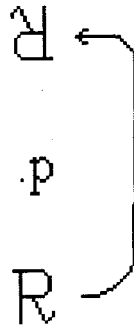


Figure 2

A **translation** is a parallel displacement with a specified magnitude. It has no invariant point since it moves every point. Figure 3 illustrates a translation.

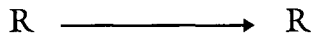


Figure 3

A **reflection** fixes all the points on a line called the mirror. Figure 4 illustrates a reflection about line n .

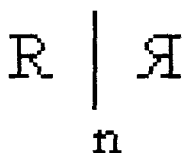


Figure 4

A **glide reflection** consists of a reflection in a line l and a translation along the same line. Figure 5 illustrates this:

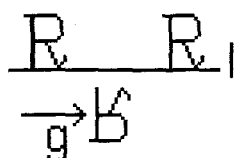


Figure 5

The set $I(\mathcal{R}^2)$, of all such isometries of the plane, together with the operation $\bullet$, denoting the composition of transformations, can easily be shown to form a group. Given two isometries a and b , we can denote the composition, called the product, by $a \bullet b$, which is interpreted as “ b followed by a .” For example the product, $\alpha \bullet \beta$ of two reflections, α and β in parallel lines is a translation, t , as shown in Figure 6.

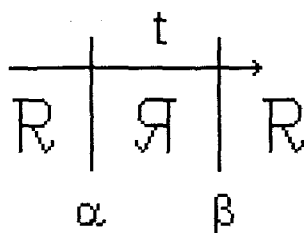


Figure 6

Moreover, a rotation is a product of two reflections in intersecting lines, while a glide reflection is a product of a reflection and a translation parallel to the axis of reflection. The translation is called the glide vector, $\mathbf{g}$, of the glide reflection. From these, we can show that the product of any two isometries is also an isometry.

Since isometries are functions or mappings, we can conclude that the operation is associative.

The unique transformation which leaves every point fixed is called the **identity transformation**, and is denoted by $\mathbf{i}$. Note that $[\mathbf{i} \bullet \mathbf{a}](P) = [\mathbf{a} \bullet \mathbf{I}](P)$, for every point P on the plane. Thus, $\mathbf{i}$ is the identity element of $I(\mathcal{R}^2)$ for the operation $\bullet$.

It can also be shown that every isometry has an inverse. For example, the inverse of a reflection $\mathbf{s}$ is itself, that is $\mathbf{s} \bullet \mathbf{s}(P) = P$. Thus, $\mathbf{s} \bullet \mathbf{s} = \mathbf{i}$. The inverse of a half-turn is itself, while that of a translation $\mathbf{t}$, of a given magnitude in one direction is a translation of the same magnitude in the opposite direction, denoted by $-\mathbf{t}$. The inverse of a glide reflection in a line l , with glide vector $\mathbf{g}$, is also a glide reflection in the same line l whose glide vector is in the opposite direction, denoted by $-\mathbf{g}$. Thus, the above discussion makes us conclude that the set of all isometries of a plane forms a group.

The subgroups of $I(\mathcal{R}^2)$ may be finite or infinite. A familiar infinite subgroup of $I(\mathcal{R}^2)$ is the subgroup of all translations. Another infinite subgroup is a **frieze group**. A frieze group with center c is a group of isometries that fix the given line c and whose translations form an infinite cyclic group. This means that each frieze group forms patterns which repeat in one direction. Even if each frieze group is infinite, each has a finite number of isometries $\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n$ such that every element of the group is of the form $\alpha_1^{k_1} \bullet \alpha_2^{k_2} \bullet \alpha_3^{k_3} \bullet \dots \bullet \alpha_n^{k_n}$, where k_i is a nonnegative integer, for each positive integer i . We call such isometries as the **generators** of the group. Table II in the text (p.25) presents the generators of the identified seven frieze groups.

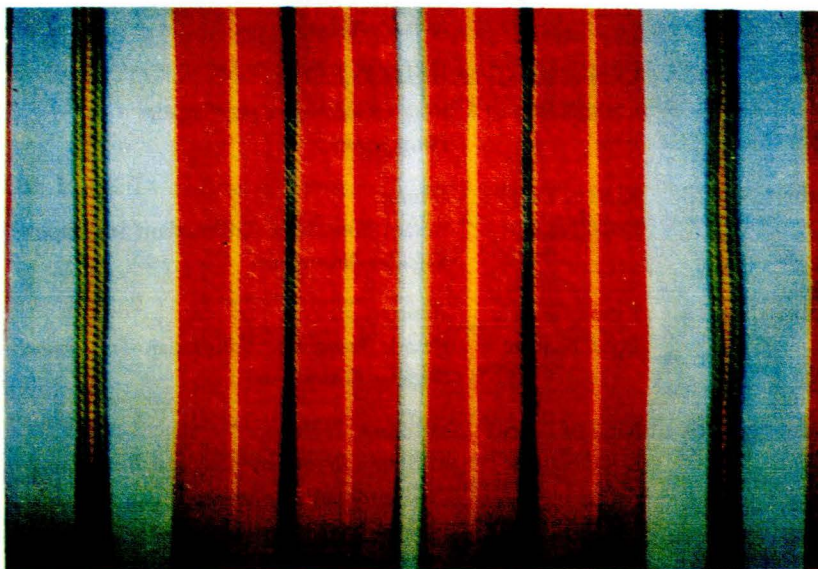
APPENDIX C

PHOTOGRAPHS OF THE KANKANA-EY FABRICS
AND THEIR LOCAL NAMES

Fabric No. 1	Kind of Fabric:	<i>Ules or Tapis</i>
	Local Names:	Besao and Guinzadan: <i>Lineyeg/Lanlan</i>
		Sagada: <i>Kinayan</i>
Fabric No. 2	Kind of Fabric:	<i>Ules</i>
	Local Names:	Besao: <i>Ginawa</i> ; Guinzadan: <i>Tinken</i>
		Sagada: <i>Innagin</i>
Fabric No. 3	Kind of Fabric:	<i>Ules or Tapis</i>
	Local Names:	Besao, Guinzadan and Sagada:
		<i>Kinulibangbang</i>
Fabric No. 4	Kind of Fabric:	<i>Ules or Tapis</i>
	Local Names:	Besao, Guinzadan and Sagada:
		<i>Pinaggagan</i>
Fabric No. 5	Kind of Fabric:	<i>Ules or Tapis</i>
	Local Names:	Besao, Guinzadan and Sagada:
		<i>Pinaggagan</i>
Fabric No. 6	Kind of Fabric:	<i>Ules or Tapis</i>
	Local Names:	Besao, Guinzadan and Sagada:
		<i>Pinaggagan</i>
Fabric No. 7	Kind of Fabric:	<i>Ules or Tapis</i>
	Local Names:	Besao and Sagada: <i>Kinain</i> ;
		Guinzadan: <i>Kinautan</i>
Fabric No. 8	Kind of Fabric:	<i>Ules or Tapis</i>
	Local Names:	Besao, Guinzadan and Sagada:
		<i>Pinaggagan</i>
Fabric No. 9	Kind of Fabric:	<i>Ules or Tapis</i>
	Local Names:	Besao and Sagada: <i>Kinain</i>
		Guinzadan: <i>Kinautan</i>
Fabric No. 10	Kind of Fabric:	<i>Ules or Tapis</i>
	Local Names:	Besao and Sagada: <i>Kinain</i>
		Guinzadan: <i>Kinautan</i>

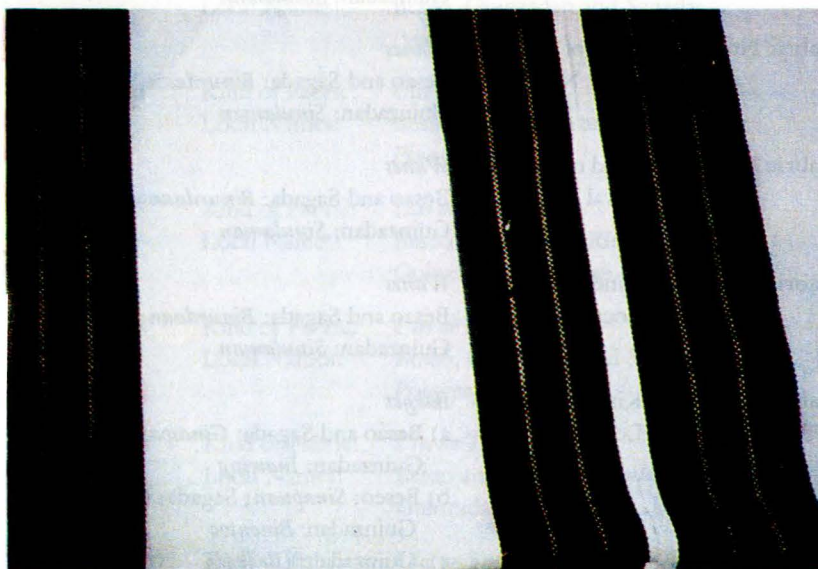
Fabric No. 11	Kind of Fabric:	<i>Ules or Tapis</i>
	Local Names:	Besao, Guinzadan and Sagada: <i>Kinulibangbang</i>
Fabric No. 12	Kind of Fabric:	<i>Ules or Tapis</i>
	Local Names:	Besao, Guinzadan and Sagada: <i>Pinagpagan</i>
Fabric No. 13	Kind of Fabric:	<i>Wanes</i>
	Local Names:	Besao: <i>Binnalikes</i> ; Guinzadan: <i>Sinulaman</i> Sagada: <i>Binurdaan</i>
Fabric No. 13	Kind of Fabric:	<i>Wanes</i>
	Local Names:	Besao: <i>Binnalikes</i> ; Guinzadan: <i>Sinulaman</i> Sagada: <i>Binurdaan</i>
Fabric No. 14	Kind of Fabric:	a) <i>Wanes</i> ; b) <i>Bakget</i>
	Local Names:	a) Besao: <i>Binnalikes</i> ; Sagada: <i>Binurdaan</i> ; Guinzadan: <i>Sinilaman</i> b) Besao: <i>Sinapuan</i> ; Sagada: <i>Golinan</i> ; Guinzadan: <i>Binontoc</i>
Fabric No. 15	Kind of Fabric:	<i>Wanes</i>
	Local Names:	Besao and Sagada: <i>Binurdaan</i> ; Guinzadan: <i>Sinulaman</i>
Fabric No. 16	Kind of Fabric:	<i>Wanes</i>
	Local Names:	Besao and Sagada: <i>Binurdaan</i> ; Guinzadan: <i>Sinulaman</i>
Fabric No. 17	Kind of Fabric:	<i>Wanes</i>
	Local Names:	Besao and Sagada: <i>Binurdaan</i> ; Guinzadan: <i>Sinulaman</i>
Fabric No. 18	Kind of Fabric:	<i>Wanes</i>
	Local Names:	Besao and Sagada: <i>Binurdaan</i> ; Guinzadan: <i>Sinulaman</i>
Fabric No. 19	Kind of Fabric:	<i>Bakget</i>
	Local Names:	a) Besao and Sagada: <i>Ginaspala</i> ; Guinzadan: <i>Inaring</i> b) Besao: <i>Sinapuan</i> ; Sagada: <i>Goligan</i> ; Guinzadan: <i>Binontoc</i> c) Guinzadan: (<i>Bedbed</i>)
Fabric No. 20	Kind of Fabric:	<i>Bakget</i>
	Local Names:	Besao: <i>Sinapuan</i> ; Sagada: <i>Goligan</i> ; Guinzadan: <i>Binonto</i>

Appendix III. PHOTOGRAPHS OF THE KANKANA-EY FABRIC AND THEIR LOCAL NAMES



Kind of Fabric: *ules* or *Tapis*

Local Names: Besao and Guinzadan: *Lineyes/Lamlan* Sagada: *kinayam*



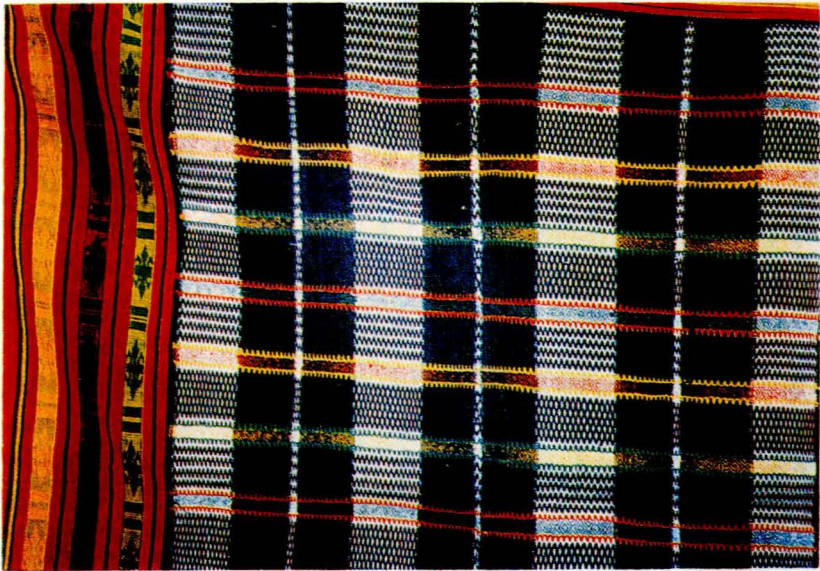
Kind of Fabric: *ules*

Local Names: Besao: *Ginawa* Sagada: *Innangin* Guinzadan: *Timken*



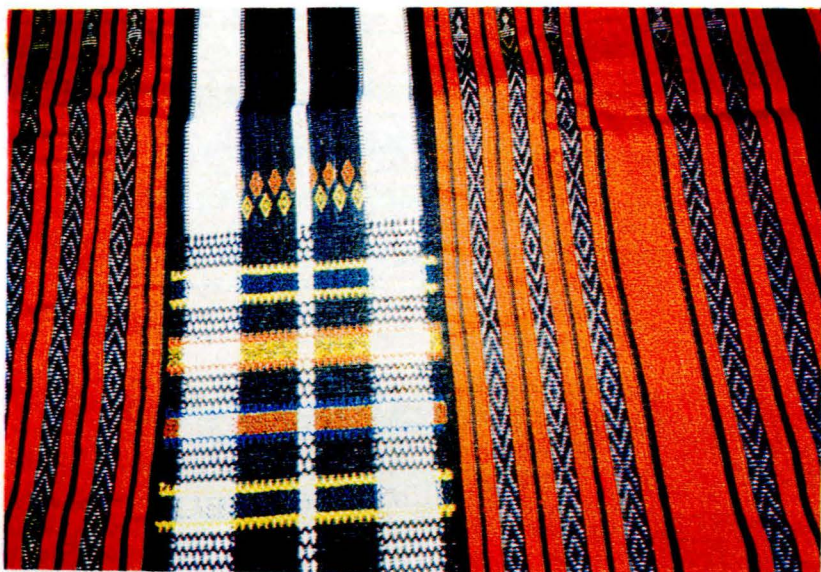
Kind of Fabric: *Ules or Tapis*

Local Name: Besao, Sagada and Guinzadan. *Kinulibangbang*



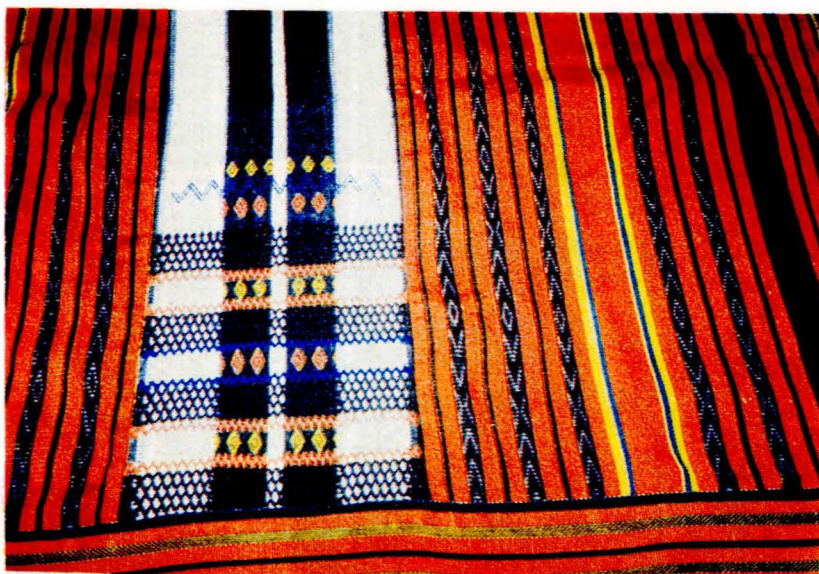
Kind of Fabric: *Ules or Tapis*

Local Name: Besao, Sagada and Guinzadan: *Pinagpagan*



Kind of Fabric: *Ules or Tapis*

Local Name: Besao, Sagada and Guinzadan: *Pinagpagan*



Kind of Fabric: *Ules or Tapis*

Local Name: Besao, Sagada and Guinzadan: *Pinagpagan*



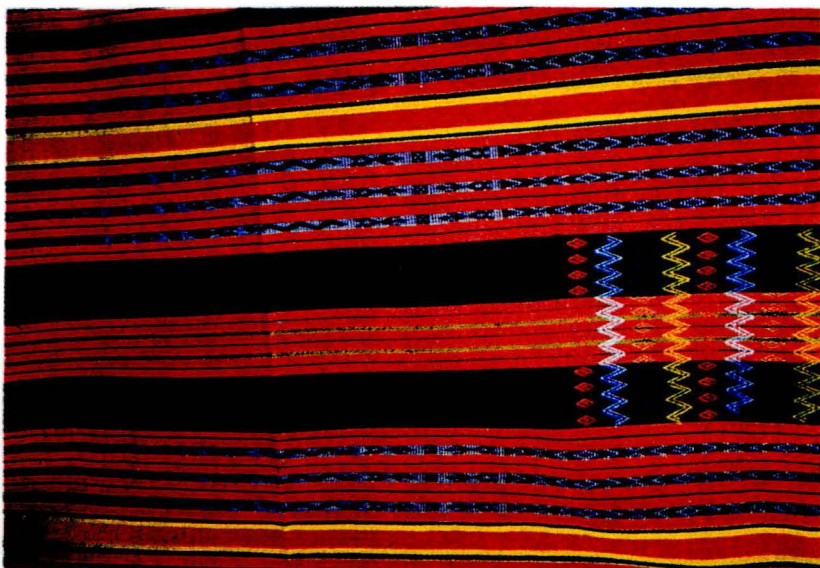
Kind of Fabric: *Ules or Tapis*

Local Names: Besao and Sagada: *Kindin* Guinzadan: *Kinautan*



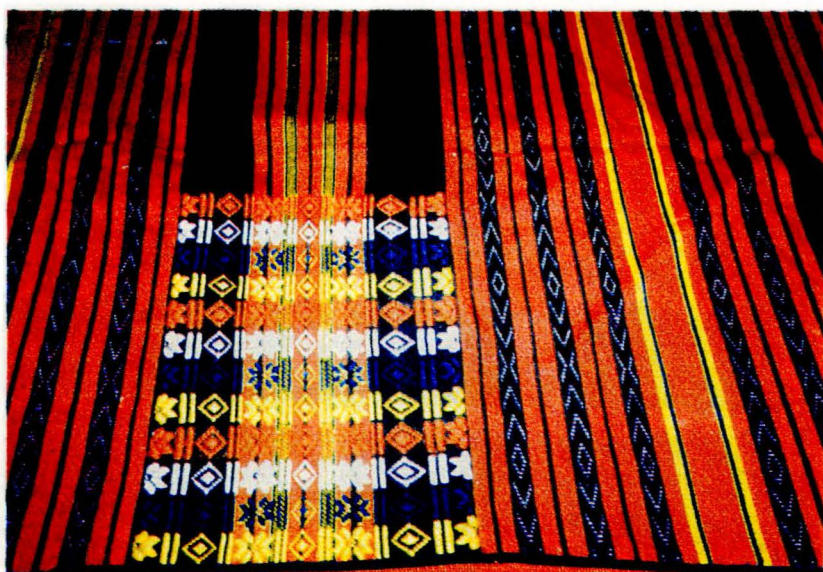
Kind of Fabric: *Ules or Tapis*

Local Name: Besao, Sagada and Guinzadan: *Pinagpagan*



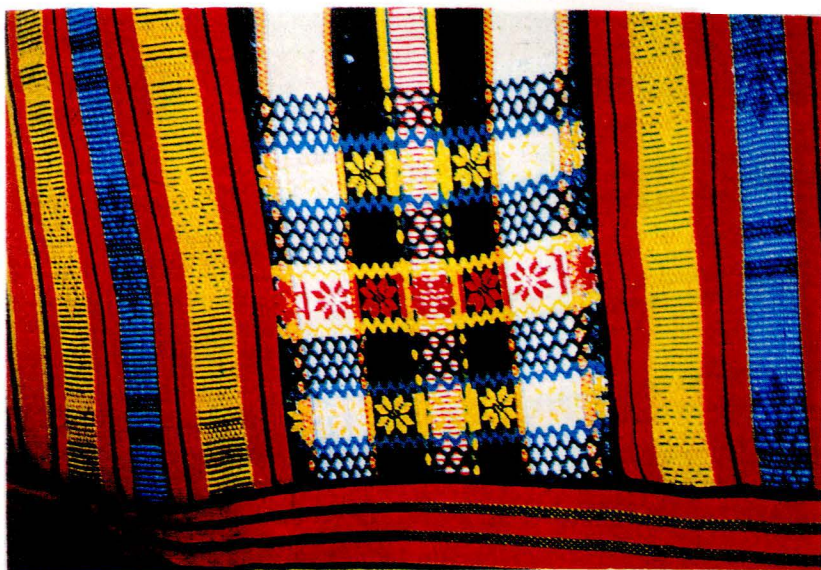
Kind of Fabric: *Ules or Tapis*

Local Name: Besao, Sagada: *Kinain* Guinzadan: *Kinautan*



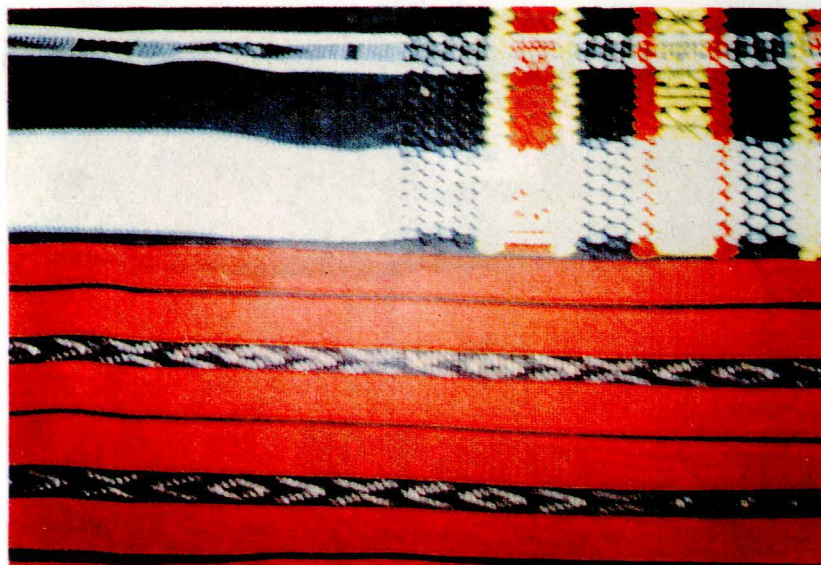
Kind of Fabric: *Ules or Tapis*

Local Names: Besao and Sagada: *Kinain* Guinzadan: *Kinautan*



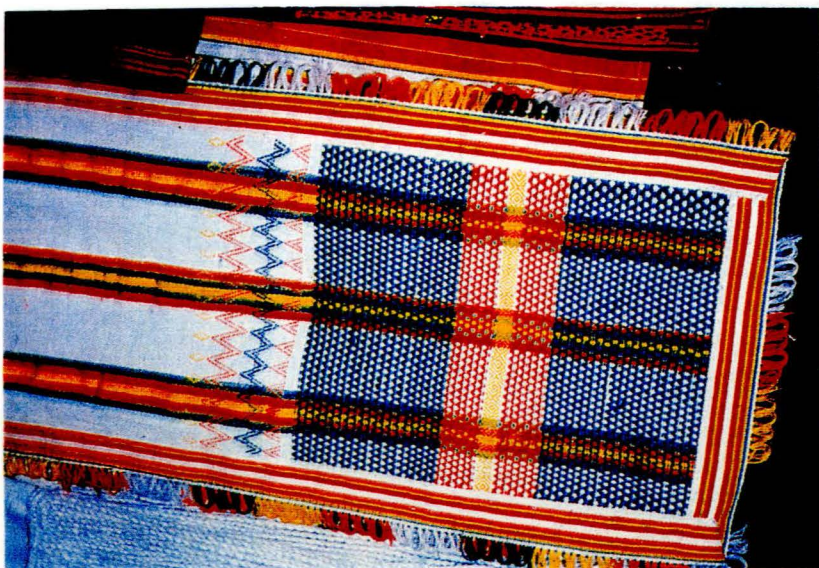
Kind of Fabric: *Ules or Tapis*

Local Name: Besao, Sagada and Guinzadan: *Kinulibangbang*



Kind of Fabric: *Ules or Tapis*

Local Name: Besao, Sagada and Guinzadan: *Pinagpagan*



Kind of Fabric: *Wanes*

Local Names: Besao: *Binnalikes* Sagada: *Binurdaan* Guinzadan: *Sinulaman*



Kind of Fabric: a: *Wanes* b: *Bakget*

Local Names: a: Besao: *Binnalikes*, Sagada: *Binurdaan*, Guinzadan: *Sinulaman*

b: Besao: *Sinapuan*, Sagada: *Golongan*, Guinzadan: *Binontoc*



Kind of Fabric: *Wanes*

Local Names: Besao and Sagada: *Binurdaan* Guinzadan: *Sinulaman*



Kind of Fabric: *Wanes*

Local Names: Besao and Sagada: *Binurdaan*

Guinzadan: *Sinulaman*



Kind of Fabric: *Wanes*

Local Names: Besao and Sagada: *Binurdaan* Guinzadan: *Sinulaman*



Kind of Fabric: *Wanes*

Local Names: Besao and Sagada: *Binurdaan* Guinzadan: *Sinulaman*



Kind of Fabric: *Bakget*

Local Names: a.) Besao & Sagada: *Ginaspala*, Guinzadan: *Inawing*
 b.) Besao: *Sinapuan*, Sagada: *Golingán*, Guinzadan: *Binontac*
 c.) Guinzadan: (*Bedbed*)

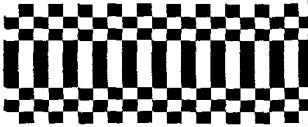
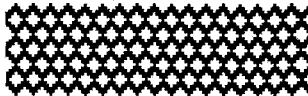


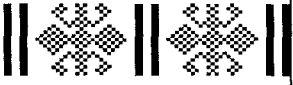
Kind of Fabric: *Bakget*

Local Names: Besao: *Sinapuan*, Sagada: *Golingán*, Guinzadan: *Binonta*

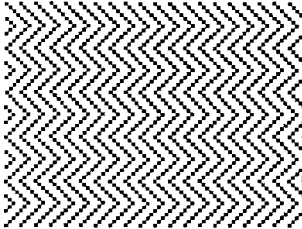
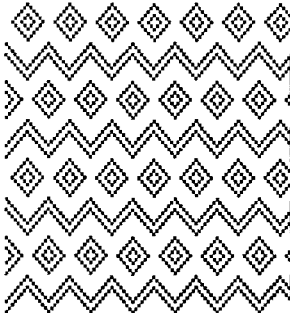
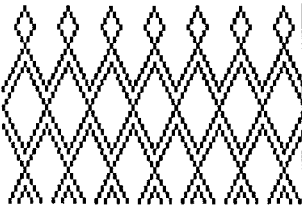
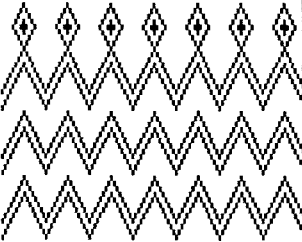
APPENDIX D

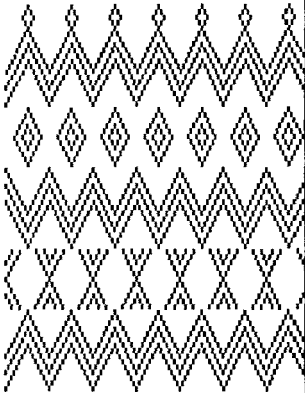
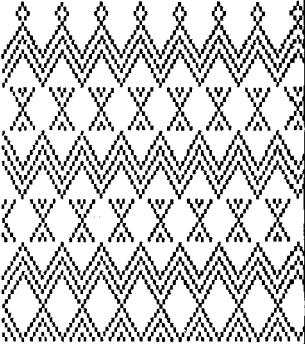
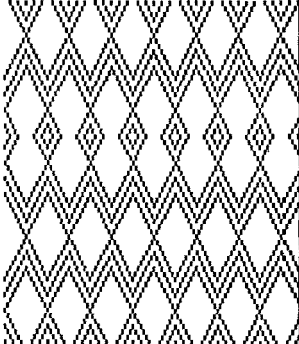
CLASSIFICATION ACCORDING TO FRIEZE GROUP
STRUCTURES OF THE DERIVED PATTERNS FROM THE
KANKANA-EY FABRICS

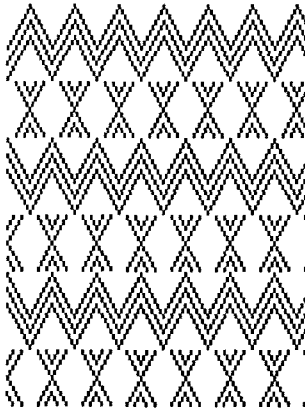
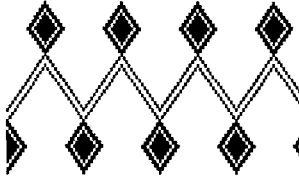
PATTERNS	F_1^1	FRIEZE F_1^2	GROUP F_2^1	F_2^2
1 				X
2 			X	
3 		X		
4 			X	

5				X	
6				X	
7				X	
8				X	
9				X	
10				X	

11				X	
12				X	
13				X	
14				X	
15				X	
16				X	

17		X			
18			X		
19			X		
20			X		

21			X		
22			X		
23			X		

24 		X		
25 				X
26 				X
27 			X	
28 			X	

APPENDIX E

TRANSCRIPTIONS OF GONG MUSIC
PLAYED DURING WEDDINGS

TAKIK ENSEMBLE

SAGADA, MOUNTAIN PROVINCE

SOLIBAO

TAKIK

PINGSAN

PINGSAN

SONOB

Legend :

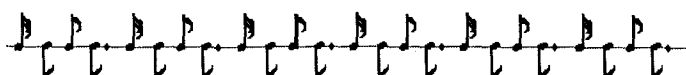
- x - muted at the inside part of the gangsa
- o - sound not stopped, let ring
- - sound is stopped on the surface with the stick

BANGUITAN, BESAO, MOUNTAIN PROVINCE
 TRANSCRIBED BY JONAS BAES
 MAY 1995

TAKIK ENSEMBLE

♩ = c.a 132

SOLIBAO



TAKIK



PINGSAN

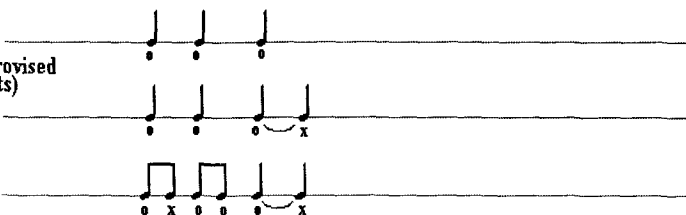


ATOMBA



SONOB

(solo or improvised parts)



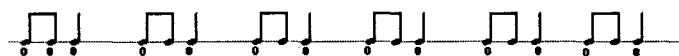
Legend:

- x - muted at the indside part of the gong
- - stopped/muted with the use of a stick
- o - "ringing tone" (sound is left to vibrate)

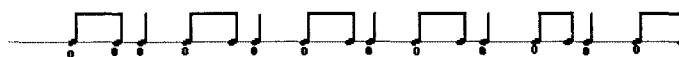
BALANGBANG ENSEMBLE

♩ = c.a. 126

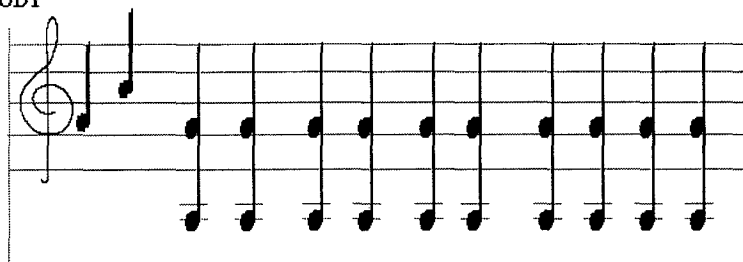
ATOMBA



PINGSAN

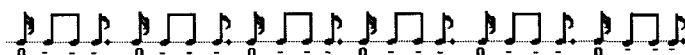


TOKTOK

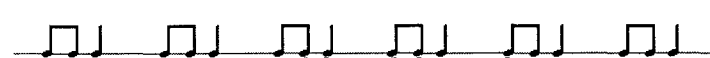
RESULTANT
MELODY**PINANYOWAN ENSEMBLE**

♩ = c.a. 126

PIYAK



SONOB



TOKTOK



TAMBOL



Legend :

♩ = stick/hand dampened when gong is struck
(slight ring tone heard)

♩ = ringing tone (sound is left to vibrate)

APPENDIX F

ADDITIONAL FEATURES OF THE KANKANA-EY KINSHIP SYSTEM

The family is the basic unit of the Kankana-ey society. It consists of the husband and wife and their children. The Kankana-ey people are monogamous and marriage is generally permanent when the couple have offspring. They are also exogamous. They can choose to marry fellow Kankana-eyes within their village or community, or those from another village or community. They are also free to forge marriage ties with non-Kankana-eyes.

Traditional courtship and marriage practices among the Kankana-ey have undergone changes because of outside influences. Here we present these traditional practices as recalled by our Kankana-ey informants.

Traditional Courtship

Older boys and unmarried males traditionally stay and sleep in the *at-ato* or *dap-ay*, while young unmarried women stay and sleep in a designated dormitory called *ebgan*. Traditional courtship starts with the boy visiting the girl in the *ebgan* usually at night. The boy expresses his feelings and sentiments through songs called *day-eng*. If the boy cannot express himself, a companion (sort of an intermediary) called *is-et* will be the one to convey the boy's intentions.

After some time of knowing and even being intimate with one another, the couple may decide to marry. Both then inform the parents of their decision. It is customary that the young man render service at the woman's house for seven or possibly thirty days. One of his works is gathering *dok-ong* or firewood. The woman in turn gathers camote leaves and vegetables and takes it to the man's house. The parents may also exchange food items. This reciprocity between them signifies the acceptance or approval of the impending relationship of their children..

If the man secures the approval of the woman's parents, the parents of both parties then perform the *og-ogbo*. The *og-ogbo* formally proclaims that the young couple are engaged. After that, the wedding is set at the woman's house.

Traditional Wedding

Wedding and ritual expenses are the primary considerations during the nuptial arrangement and preparation. Brideprice, which is no longer important today as it was before, consists of land and other valuables (like precious beads from heirloom), depending on the wealth of the man's family or the demands of the bride's family. However, the prevailing practice today is that both families shoulder all expenses that will be incurred.

The marriage ceremony or *dawak* and its accompanying rituals start only after the village people have built a house where the couple are to be wed. After the house is built, the *begnas di dadawak* begins. It is a two-day festival where animals such as pigs and chickens are sacrificed in a designated *dap-ay* or private house called *daw-es*. Through these sacrifices, the community elders and the families seek the gods' blessings for the newly-wed couple.

Impact of Changes

The elders of the community have identified some changes in the traditional courtship and marriage practice. For one, there is no longer the *ebgan* where formal courtship takes place. Instead, the suitor now goes directly to the woman's house and proposes. In some cases, a couple may decide to get married with minimal involvement of parents. Marriages contracted by parents are now things of the past.

Traditional marriage rituals are also disappearing now that marriage is performed by the Christian Church. Some Kankana-ey mix traditional practices with "modern" ways of marriage. They still perform some rituals and offer sacrifices, but with some deviations from what is strictly required by tradition.

The elders accept with skepticism these changes brought about by outside influences, like the influx of Christianity and formal education. The elders lament over the changes and modifications. They are concerned over the prospect that the Kankana-ey marriage customs and traditions might eventually be forgotten.

Kankana-ey Kinship Terminologies

The table below presents Kankana-ey terminologies classified according to terms of address and terms of reference.

Kankana-Ey Kinship Terms

A. Consanguineal (By Birth)

Nuclear Family

<u>Relationship to Ego</u>	<u>Reference</u>	<u>Address</u>
parents	ama/ina	ama/ina
father	ama	ama
mother	ina	ina
child/children	anak	
first/oldest child	pangpangu/iyun-a	nickname
middle child	gawa-an	nickname
last/youngest child	kutit/kimut	nickname
son	anak/anak ay lalaki	nickname
daughter	anak/anak ay babai	nickname
sibling	besat	nickname
older brother/sister	iyun-a(/ay lalaki)	nickname
	iyun-a(/ay babai)	nickname
younger brother/sister	ina-udi(/ay lalaki)	nickname
	ina-udi(/ay babai)	nickname
spouse	asawa	
husband	ama-n + 1st child's name/	amana(one child)
wife	ina-n + 1st child's name	inana(one child)
	asawa ay babai	inada(many children)

Extended Family

<u>Relationship to Ego</u>	<u>Reference</u>	<u>Address</u>
ancestors	ap-apu	ap-apu
grandparents	alapu/aalapu	
grandfather	alapu/alapu ay lalaki	alapu
grandmother	alapu/alapu ay babai	alapu
uncle	alita-o	alita-o
aunt	ikit ikit	
cousin	pingsan/kassinsin	nickname
first	pingsan/pingpingsan	nickname
second	pidpidua	nickname
third	pitpitlo	nickname
fourth	pippipat	nickname
nephew	kaanakan(/ay lalaki)	nickname
niece	kaanakan(/ay babai)	nickname
grandchild(ren)	apu nickname	
great grandchild(ren)	pidwun di apu	nickname
relatives	agi	

B. Affinal (By Marriage)

<u>Relationship to Ego</u>	<u>Reference</u>	<u>Address</u>
parents-in-law	katugangan	
father-in-law	katugangan(/ay lalaki)	ama
mother-in-law	katugangan(/ay babai)	ina
child-in-law	inapu	nickname
son-in-law	inapu(/ay lalaki)	nickname
daughter-in-law	inapu(/ay babai)	nickname
brother-in-law	kasa-ud	nickname
sister-in-law	aydu	nickname
husband/wife of si/br-in-law	abirat	nickname

A Kankana-ey addresses relatives of his own generation and of the younger generations by their names or nicknames. He addresses his parents by *ama*(father) and *ina*(mother). When he speaks to his grandparents or members of the older generation, he uses *apu* or *ap-apu*. When he speaks to a group or crowd whose members are of his generation or of the generations after him, he calls to them *agi*. If the group or crowd is composed of elders, he calls to them *ap-apu*.

A Kankana-ey refers to his parents as *ama* and *ina*, to his grandparents and elders as *apu*, to his relatives in his and in the minus generations as *agi*. He refers to cousins by the general term *pingsan* or *kasinsin*. But when he becomes specific, he uses descriptive terms, like *pingsan* for first-degree cousins, *pidpidua* for second degree cousins and *pitpitlu* for third degree cousins. He refers to his children as *anak* and to his grandchildren as *apu*. In particular, he uses *apu* when referring to immediate grandchildren, *pidwu di apu* for great grandchildren and *pitlu di apu* for great great grandchildren.

An interesting practice among the Kankana-ey is describing a person in relation to his or her first-born child. For example, a couple in one site are called in the community as *aman Ligaya* (Ligaya's father) and *inan Ligaya* (Ligaya's mother) because their first child is a girl named Ligaya. This practice is called **teknonymy**.

APPENDIX G

THE EIGHT AXIOMS OF HARRISON WHITE

Below are the eight axioms of Harrison White in his mathematical analysis of kinship. The axioms are taken from *White's An Anatomy of Kinship* (1963) and are reprinted here verbatim.

1. The entire population of the society is divided into mutually exclusive groups, which we call clans. The identification of a person with a clan is permanent.
2. There is a permanent rule fixing the single clan among whose women the men of a given clan must find their wives.
3. By rule 2, men from two different clans cannot marry women of the same clan.
4. All children of a couple are assigned to a single clan, uniquely determined by the clans of their mother and father.
5. Children whose fathers are in different clans must themselves be in different clans.
6. A man can never marry a woman of his own clan.
7. Each person in the society has some relative by marriage and descent in each other clan: i.e., the society is not split into groups not related to each other.
8. Whether two people are related by marriage and descent links are in the same clan depends only on the kind of relationship, not on the clan to which either one belongs.

Teofina A. Rapanut (Project Leader/ Team Leader - Weaving Team) is Associate Professor of Mathematics and the current Program Head of the graduate program Master of Arts (Mathematics Education), University of the Philippines College Baguio. She obtained her Ph.D. (Mathematics) from the University of the Philippines Diliman.

Wilfredo V. Alanguí (Team Leader, Kinship Team) is Assistant Professor of Mathematics and current Discipline Head of Mathematics, Division of Natural Sciences & Mathematics, University of the Philippines College Baguio. He is also the Coordinator of the UPCB College Extension Program/*Ugnayan ng Pahinungod*. He obtained his M.S (Mathematics) at the University of the Philippines Diliman.

Henry N. Adorna (Team Leader, Music Team) is Instructor in Mathematics and currently pursuing a graduate degree in M.S. (Mathematics) at the University of the Philippines Diliman.

Reamar Eileen R. Sales (Weaving Team member) is Assistant Professor of Mathematics. She obtained her M.S. (Mathematics) from the University of the Philippines Diliman and is currently pursuing a Ph.D. (Mathematics) program at the University of the Philippines Diliman.

Madelyn N. Ragasa (Weaving Team member) is Instructor in Mathematics. She obtained her M.A. (Mathematics Education) from the University of the Philippines College Baguio.

Avelino P. Bucaoto (Kinship Team member) is Assistant Professor of Mathematics and Statistics. He obtained his M.A.T. (Mathematics) and M.S. (Statistics) from the University of the Philippines Diliman.

Arlano R. Aquino (Kinship Team member) is Instructor in Mathematics. He obtained his B.S. (Mathematics) *cum laude* from the University of the Philippines College Baguio. He is currently pursuing a Master of Arts (Mathematics Education) at the University of the Philippines College Baguio.

Reynaldo P. Rimando (Music Team member) is Assistant Professor of Mathematics. He obtained his M.A.T. (Mathematics) from the University of the Philippines Diliman.

Edna M. Nazaire (Music Team member) is Instructor in Mathematics. She obtained her B.S. (Mathematics) *cum laude* from the University of the Philippines College Baguio. She is currently pursuing a Master of Arts (Mathematics Education) at the University of the Philippines College Baguio.

Darlina N. Bautista, Feliciano T. Vergara, Jr. were Research Assistants for the project.

The Algebra of the Weaving Patterns, Music and Kinship System of the Kankana-ey of Mountain Province

is a pioneering study on Philippine ethnomathematics, or the mathematics of a people's culture and community life.

Focusing on certain features of the indigenous ways of the Kankana-ey of Cordillera, the study shows that mathematics is concretely embedded and operative in a people's culture. This has deep and far-reaching implications on three fronts: on our mathematical and pedagogical knowledge, on making such knowledge more accessible and relevant to the people, and on developing more responsive mathematics curricula in formal and non-formal education.

